PHY 115L
WAVE MOTION AND OPTICS

LAB MANUAL FOR PHY 315

Department of Physics
University of Texas at Austin
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-M.E.L. Oakes (2003)

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Contents

Introduction

This laboratory consists of experiments on wave motion. The purpose of this lab is to get you familiar with phenomena that you will encounter in nature and possibly even in the lectures. With the brief theoretical guidance provided in these notes, you can isolate and observe a variety of important properties of waves. This can be extremely useful in your attempts to analyze and understand more complex phenomena and systems.

As a rule most scientists believe that theory should rarely stray too far from experiment — there are exceptions, but they are rare. Whether experiments likewise should also be restricted to remaining near existing theories is often heatedly debated. Most scientists, though, would agree that one cannot separate good experiments from the theory motivating it.

While the notes accompanying each lab make it self-contained, the theory is incomplete. Remember that Wikipedia is your friend, as is your lecture textbook. Everyone can carry out the procedures outlined, but, in order to gain from the lab, you must do this intelligently. Do not isolate the laboratory from the lecture, even when the two are not in sync.

To borrow an analogy from another science, after a walk through a forest accompanied by a trained botanist, there is no doubt as to who *learned the most* versus who was *merely entertained*. Combining your knowledge of theory from the lecture and your intuition from the laboratory is the path toward being the former. If you do this, you will come away from the course having added new and powerful tools for solving the difficult problems encountered in your careers.

Course Details

Bring your laboratory notebook and your manual to the first class.

It is important that you keep a laboratory notebook in which you record *all* methods, data, and observations. Be as verbose as you feel necessary, and write in it what you would need if you were asked six months later to reconstruct and present a report on the experiments using only this manual and your notebook. Never erase anything, even if it is wrong.

The manual has many questions distributed throughout the discussion and procedure sections. It is important that you answer each question before proceeding. These questions are there to ensure that you are on the right track. If you are having problems answering a question, then ask the instructor.

Much of the equipment you are using is very expensive, so you will be expected to handle it with the care of a professional scientist. Notify the instructor of any equipment failures.

The administrative details of the course will be discussed during the first laboratory class.

You will start your first experiment at the first lab meeting.

EXPERIMENT 1

Vibrations of Strings

1.1 Equipment Required

1. Spring
2. Timer
3. Meter Stick
4. Sonometer
5. Sound Sensor
6. Computer
7. Weights
8. 750 Interface Box

1.2 Introduction

Most of us have on occasion observed systems undergoing oscillations. This may have been a pendulum in a clock, a boat in which you were sitting, the front end of your car, a child's jump rope, or the surface of the lake around a tossed stone. A little reflection might cause you to distinguish between such events in the following way:

1. Those which are disturbed and then left to their own devices
2. Those which continue to be subjected to a disturbing force

The first of these we call free oscillations and not surprisingly the second we call forced oscillations. Examples of 1: A plucked guitar string, a punching bag struck once, and a trampoline you have vacated. Examples of 2: A child's jump rope being thrown, the clock pendulum (the weights and escapement provide the driving force), and radio waves radiated by currents in an antenna.

Question 1.1 List several examples in each category.

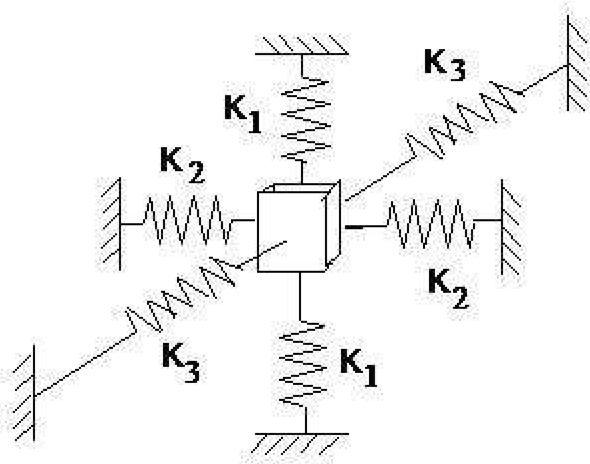


Figure 1.1: 3 dimensional harmonic oscillator.

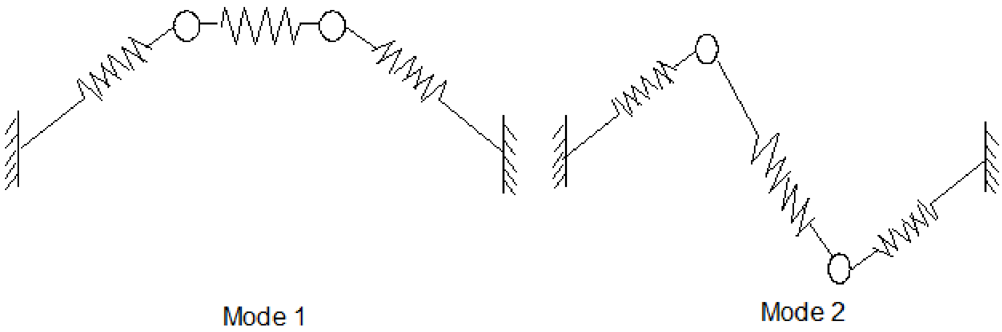


Figure 1.2: Normal modes for 2 particles.

1.3 Free Oscillations

Let us begin with small amplitude free oscillations. The classic example is a spring-mass system which, you might recall, exhibits simple harmonic oscillation at a characteristic frequency $\omega_0^2 = k/m$ where k is the spring constant and m is the mass attached to the spring.

For this one particle with motion constrained to a line there is one frequency. If the particle were free to move in three directions, i.e., had three degrees of freedom, we would find, for small displacements, its motion to be a linear superposition of harmonic motions with three frequencies (Fig. 1.1). You will learn in the lectures that the number of characterizing frequencies of a linear system is equal to the number of degrees of freedom present.

These characteristic frequencies are sometimes called *eigenfrequencies* or *normal mode frequencies.* To get a complex system to oscillate at only one of the normal mode frequencies will obviously require that the system's motion have a very special "shape". These special shapes are called *normal modes* (*eigenmodes* or characteristic modes). For a one particle spring-mass system, there is only one normal mode, the particle simply oscillates simple harmonically with an amplitude determined by its initial displacement and velocity. For two particles, coupled as in Fig. 1.2 and confined to move along vertical lines, there will be two normal modes.

Question 1.2 Which mode will have the higher frequency and why?

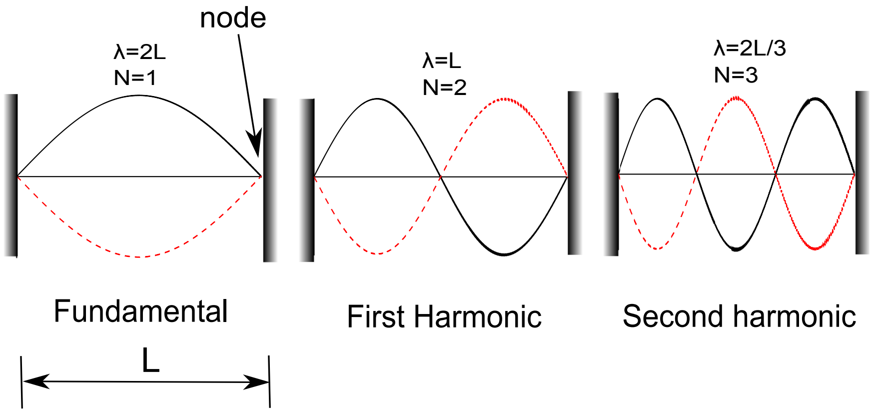


Figure 1.3: Fundamental, first and second harmonic.

1.4 Normal Modes of a Long Spring

As we increase the number of particles coupled by springs we have more normal modes. A long spring is convenient for studying normal modes of a system with many particles (or, in this case, loops). The first few normal modes (or standing waves) are sketched in Fig. 1.3. (The dotted curves show the spring one half period after the solid lines.) The shapes of the modes depend on the conditions at the end (commonly called boundary conditions).

The modes in Fig. 1.3 are for a spring with both ends fixed. These modes will clearly have different shapes if one or both ends are free to oscillate.

Let us study the transverse motion of the long helical spring while in one of its normal modes.

Since, for the modes shown, the ends are fixed, it will be necessary for your partner to hold one end fixed while you shake the other end at the frequency of the normal mode of interest. Once the mode is excited, hold your end rigid. You may have to refresh the mode periodically. Other modes you might have excited will die out quickly.

Question 1.3 Using a timer determine the frequencies of three modes. Plot the frequencies vs. some scale length for each mode (if you have difficulty deciding what is an appropriate length then talk with the TA). Make sure you do not change the equilibrium length of the spring as you excite the different modes. Why is this important?

Question 1.4 Does there appear to be a relationship between the frequency and the scale length? What is it? Give an argument for there being a connection between the scale length and the frequency.

Question 1.5 Plot the mode frequencies vs the inverse of the mode's scale length. What are the units of the slope of this curve? Find the product of each mode's frequency and its scale length. Compare this "velocity" with the measured velocity of a pulse on the spring. In measuring the pulse velocity make sure the spring has the same length as before.

Question 1.6 Compare the period of the fundamental mode with the time for a pulse to go down and return.

1.5 Free Oscillations on a String

Question: Why all this interest in these special shapes and the motion of a spring or string?

Answer: Joseph Fourier showed that the general motion of a small amplitude string is just a simple sum (linear superposition) of these very special motions or normal modes. How much of each mode is present depends on the initial shape and velocity of the string. Clearly each mode will contribute its own frequency and characteristic length to the motion and thus to the sound and shape that the string has. For a string to oscillate with only one frequency requires that only one mode be present.

Put sufficient weight on the sonometer string to provide a taut string and a pleasant sound when plucked.

Place the sound detector near the sonometer's resonance box and connect the sensor's DIN plug to the Analog Channel A on the Science Workshop Interface Box (Pasco 750)

Turn on the interface box and the computer. If not currently booted press the button on the right side of the computer for 5 sec. The user name is *115l-7322*, and the password is *115l*.

In the sensors list displayed, double-click on the "Sound Sensor" (with the ear icon). This will associate the sensor with the interface channel.

In the Display list, double-click on the Graph item. Axes should appear. Note the quantities plotted and the units. Now double-click on the graph anywhere. This should open a window to customize your plot. Select the Axes Scaling tab. This will let you adjust Min and Max Times and Unit (choose milliseconds). The Appearance tab will be useful as you proceed. Close Graph Settings window.

Excite your sound source and click the start button at the top on the main toolbar. Data should appear in real-time in the display.

To stop collecting data, go to the main toolbar and click the stop button. Explore the collected data.

The frequency content of your data can be measured using the FFT option in the displays window where you found graph earlier (FFT means Fast Fourier Transformation). Double click the FFT icon and explore the frequencies present in sound from the string.

Let us return to the string on the sonometer. Stop now and think about the string you are plucking. By putting weights on the end, what are you changing? When you pluck the string by a small amount near the middle, you give the string a shape something like Λ then you let it go.

Question 1.7 If you concentrate your attention on the small part of the string at the apex and observe how fast it returns to the center, what is likely to be the dependence of this time on the equilibrium tension in the string?

1.6 Frequency Spectrum

Pluck the string and record its pattern on the computer with the DataStudio Graph studied before, it may be quite complicated (remember however it is just a sum of normal modes. A pattern that can result from just three modes is shown in the appendix at the end of the experiment.).

Pluck the string in different positions, note how the pattern, the sound and the frequencies present change. In order to see the frequency content, use the Fast Fourier Transform (FFT) function of the DataStudio Program.

Question 1.8 Does the frequency content change based on where you pluck the wire?

1.7 Tension and Length Dependence

Question 1.9 Are there conditions under which you see a nice sinusoidal vibration on the screen? Is there a dominant frequency?

The string then is oscillating in only one mode, probably the fundamental. The early complicated pattern results from exciting additional modes such as first and second harmonics. The higher harmonics lose their energy more rapidly than the lower harmonics, thus the fundamental is the last to die out.

Question 1.10 Can you give a reason why the higher harmonics might be damped more?

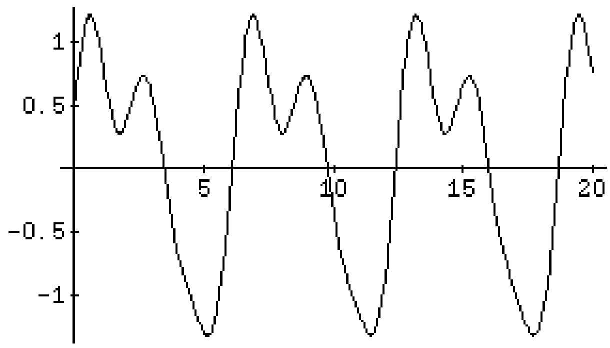
Question 1.11 Using the sound sensor and DataStudio, determine the dependence of the frequency of the fundamental mode on string tension.

Question 1.12 Using the sound sensor and DataStudio, determine the dependence of the frequency of the fundamental mode on string length.

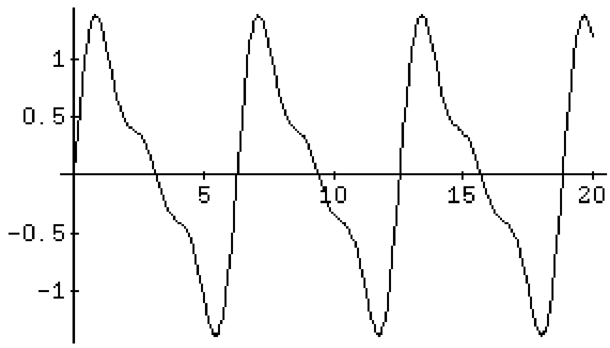
Question 1.13 Summarize what you have learned about the normal modes (i.e. standing waves) of a string.

1.8 Appendix

This is the superposition of three modes. The first has twice the amplitude of the second and the second has twice the third and the second has a phase shift: $y = 1\sin(t) + .5\sin(2t+1) + .25\sin(3t)$.



If we remove the phase shift: $y = 1\sin(t) + .5\sin(2t) + .25\sin(3t)$, we get:



Get access to Mathematica, Theorist, Maple, MathCad or other scientific analysis program and superimposed sinusoidal modes of different amplitudes and phases. Let the frequencies be integral multiples of each other.

EXPERIMENT 2

Forced Oscillations

2.1 Equipment required

1. Driven Tuning Fork
2. Power Supply
3. C-Clamp
4. Pulley
5. String
6. Weight Holder and Weights
7. Sound Sensor
8. Interface Box and Computer
9. Rubber Mallet
10. Triple Beam Balance

2.2 Introduction

You have studied the free oscillations of a string and explored the dependence of the normal mode frequencies f_n on the string's tension T, mass per unit length ρ, length L, and mode number n. The exact expression is

$$f_n = \frac{n\sqrt{T/\rho}}{2L}, \tag{2.1}$$

where f_n is in Hertz, T is in Newtons, ρ is in kg/m and L is in meters.

Now we can move from free oscillations to forced oscillations. Let us first recall the simple harmonic oscillator. In free oscillations the system had one normal mode with one normal frequency $\omega_0 = \sqrt{k/m}$. Now when one drives this oscillator at a frequency $\omega_D \neq \omega_0$ then the amplitude of oscillation is quite small. The closer the driving frequency ω_D is to ω_0, the larger the response. At $\omega_D = \omega_0$, the mass executes large amplitude motion and we say that the driver has hit the resonant frequency of the oscillator. Said another way, the normal mode frequency is the resonant frequency of the forced system.

Likewise you will show in the lectures that the standing wave frequencies of a string will be the resonant frequencies of a string being driven.

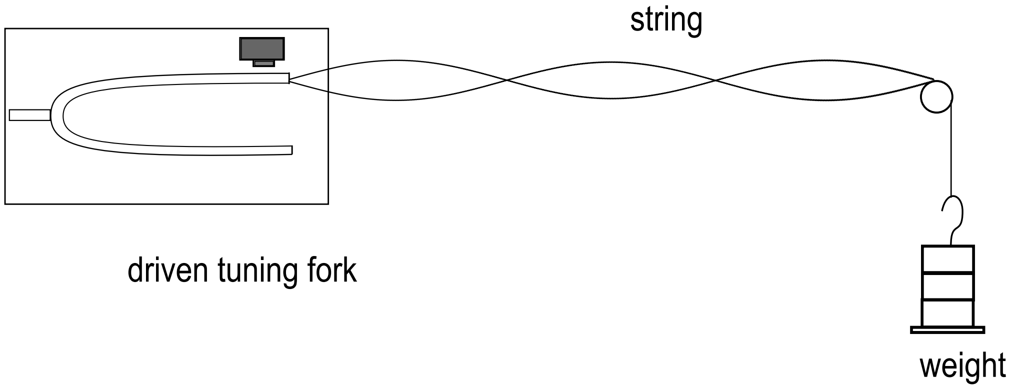


Figure 2.1: Resonance condition with $n = 3$.

2.3 Free Oscillations of a Fork

Since a tuning fork will be our driver, let us first study the behavior of a freely oscillating fork. Use sound sensor to observe the fork's motion and its frequency spectrum. Rap the fork sharply on the bottom of your shoe. Place the Sensor near one tine of a small tuning fork. Observe its waveform on the computer.

Question 2.1 Determine its dominant free oscillation frequency.

Question 2.2 What is the next most important frequency in your measured spectrum? Make sure your frequency axis extends to at least 7 times the fundamental frequency.

Question 2.3 Does the fork's motion appear to be simple harmonic?

2.4 Driven Oscillations of Fork

In order to study forced oscillations (Melde's experiment), set up the tuning fork apparatus as shown in Fig. 2.1.

The weights on the end permit us to change the tension in the string. The length may be changed by repositioning the tuning fork. Take a moment to observe how the fork is driven by the DC power supply.

Question 2.4 Record in your own words how the fork is driven.

In studying driven modes of the string, the obvious thing to do would be to get an oscillator that would permit you to change the frequency in a continuous manner. Then you could attach it to the string and vary the frequency until you hit one of the frequencies determined by Equation 2.1.

As you continued to increase the frequency you would encounter the various normal mode frequencies of the string. How would you make a variable frequency mechanical oscillator? The difficulty in building such a device forces one to take a different approach. Using a fixed frequency oscillator (the tuning fork), you can vary T or L and look for conditions for which a resonant frequency of the system matches the driving frequency.

2.4.1 Length Dependence

We first will look at the L dependence. Arrange the fork such that the string is approximately one meter long. Start the tuning fork oscillating. Put enough tension in the string (using the weights) to obtain a resonance with a value of n of 4 or 5.

The contact adjustment is touchy so you may need help from the laboratory instructor. Cleaning contacts with sandpaper sometimes helps. Now that the fork is oscillating, note what the string is doing. Starting at the pulley end of the string, gently place a pencil in contact with the string and slowly move toward the tuning fork.

Question 2.5 Record the mode number and string length for any resonance you observed. The number of loops (maxima) identifies the mode number n.

Question 2.6 Compare the shape of the string at a resonance with the shape of a string with the same mode number (n) undergoing free oscillation.

Question 2.7 Where are the nodes (zero displacement points) for the mode number you choose?

Question 2.8 What does nonresonant oscillation on the string look like? Nonresonant oscillation occurs when the driver does not match a standing wave frequency.

Question 2.9 What is the distance between the node nearest the tuning fork and the tuning fork?

Question 2.10 What happens to the node as you approach resonance?

Question 2.11 Plot your values of n vs. L and compare with the predictions of Equation 2.1. You will need to measure ρ. A triple beam balance should be available.

2.4.2 Tension Dependence

Place enough mass on the weight holder so that the mode $n = 1$ or $n = 2$ is present; now adjust the string's length until the fundamental mode ($n = 1$) is present.

Question 2.12 Determine the tensions required to produce resonances with three or four different values of n. (Will this be higher or lower tension?) Fine adjustments (in order to determine whether more or less weight is needed) can be made by pressing up or down on the weights.

Question 2.13 Plot n vs. T on a linear scale.

Question 2.14 Fit the data with the theoretically predicted curve.

Question 2.15 plot your results n vs. T on a log-log scale.

Question 2.16 What would be the advantage of a log-log plot?

If the driving frequency is close to the resonant frequency of the string you will see a slow modulation of the string, these are called *interminable beats*.

2.4.3 Losses (Sound radiation, air drag and fiber friction)

Cause the string to oscillate at a $n = 2$ or 3 resonance; observe the nodes. Note that they appear to be at rest; if this is true then nothing should change if you grasp the string at the node.

Question 2.17 What happens when you do this? Give an explanation.

EXPERIMENT 3

Velocity of Sound in Air

3.1 Equipment Required

1. Glass Tube, Stand, and Water Jug
2. C-Clamps
3. Tuning Fork
4. Meter Stick
5. Kundt's Tube
6. Rosin and Chamois Cloth

3.2 Introduction

In this experiment we look at the normal modes in a cylindrical tube containing air at atmospheric pressure. In order to conduct one of the experiments we will also excite a normal mode in a metal bar clamped in the middle but free at the ends. The oscillations are approximately longitudinal in both systems, i.e., the gas and the metal move back and forth along the tube axis.

The oscillations are in the pressure, density and velocity. We are interested in the velocity of disturbances in the medium. This velocity is directly related to the shape of a mode and its associated frequency. If we could see the air particles in the glass tube at an appropriate point in time, their density variation might be as shown in Fig. 3.1.

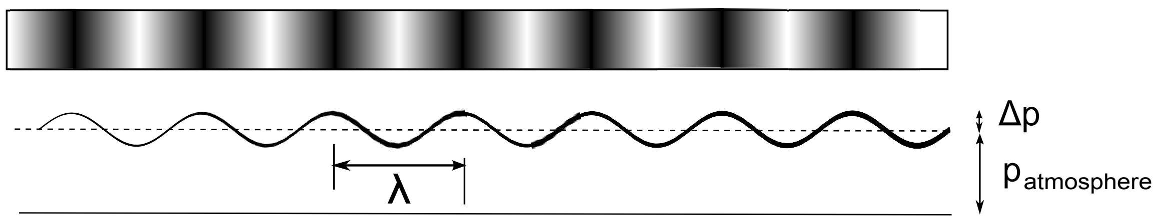


Figure 3.1: Density (upper) and pressure (lower) variation of air particles in the glass tube at an appropriate point in time. $p = p_{atmosphere} + \Delta p$ where $\Delta p = \Delta p_0 \cos(2\pi x/\lambda) \cos \omega t$

Also shown is an expression for a normal mode, note that the pressure swings above and below the atmospheric pressure in the tube. Since it is pressure differences that accelerate the particles and since the pressure gradient($\frac{\partial p}{\partial x}$) is largest at the pressure node, the velocity has a maximum at the pressure nodes. The pressure and the velocity are also ninety degrees out of phase in time. As with any normal mode analysis, it is important to specify the boundary conditions.

Question 3.1 For an enclosed tube the velocity of the particles is zero at the closed end. Can you provide any justification of this condition?

Question 3.2 What would the pressure be doing at a closed end?

Question 3.3 If we were to place a thin layer of cork dust in the bottom of the tube and excite a normal mode, how would the dust be redistributed? Support your answer.

3.3 Air Column-Tuning Fork Method for Determining Velocity of Sound in Air, C_{Air}

Consider a glass tube closed at one end and open at the other end. The column of air in this tube is capable of vibration.

Question 3.4 At the closed end we expect a node in the velocity and an antinode in the pressure (see earlier discussion). What would we expect at the open end?

A wave approaching an open end tube with $\lambda \gg$ *tube diameter* will be reflected. The pressure will undergo phase reversal — a compression reflects as a rarefaction. At the end the superposition of these two waves yields a node in the pressure. The velocity and displacement of the air will have antinodes at the open end.

As you might suspect the air just outside the tube is moving also. This additional air in motion results in the pressure node being a small distance outside. This can be corrected by computing a new effective length of the tube. This added length E is known as the *end correction* and for an unflanged circular tube is approximately 0.6 of the tube radius.

A few of the free oscillations of an air column are shown in Fig. 3.2. Three different length tubes are displayed. Displacement nodes are shown at the rigid ends and displacement antinodes at the open ends.

Thus normal modes for the air columns shown satisfy the relationship

$$L + E = n\lambda/4, \tag{3.1}$$

where L is the actual length of the column and n is an odd integer and λ is two times the measured distance between nodes.

If we can determine λ and ν in the dispersion relation we can get

$$C_{\text{air}} = \omega\lambda/2\pi \quad = \lambda\nu \tag{3.2}$$

where C is the speed of sound and ν is the frequency. We will use a tuning fork of known frequency to force the column to vibrate at that frequency.

Question 3.5 Is there only one frequency? What effect would another strong harmonic have on your results?

If the column has the correct length, then a normal mode frequency of the column will match the frequency of the driving tuning fork. We then have resonant oscillation of the air column and we should detect loud sound vibrations radiated into the room.

Question 3.6 Will you excite vibrations of the water column? Why or why not?

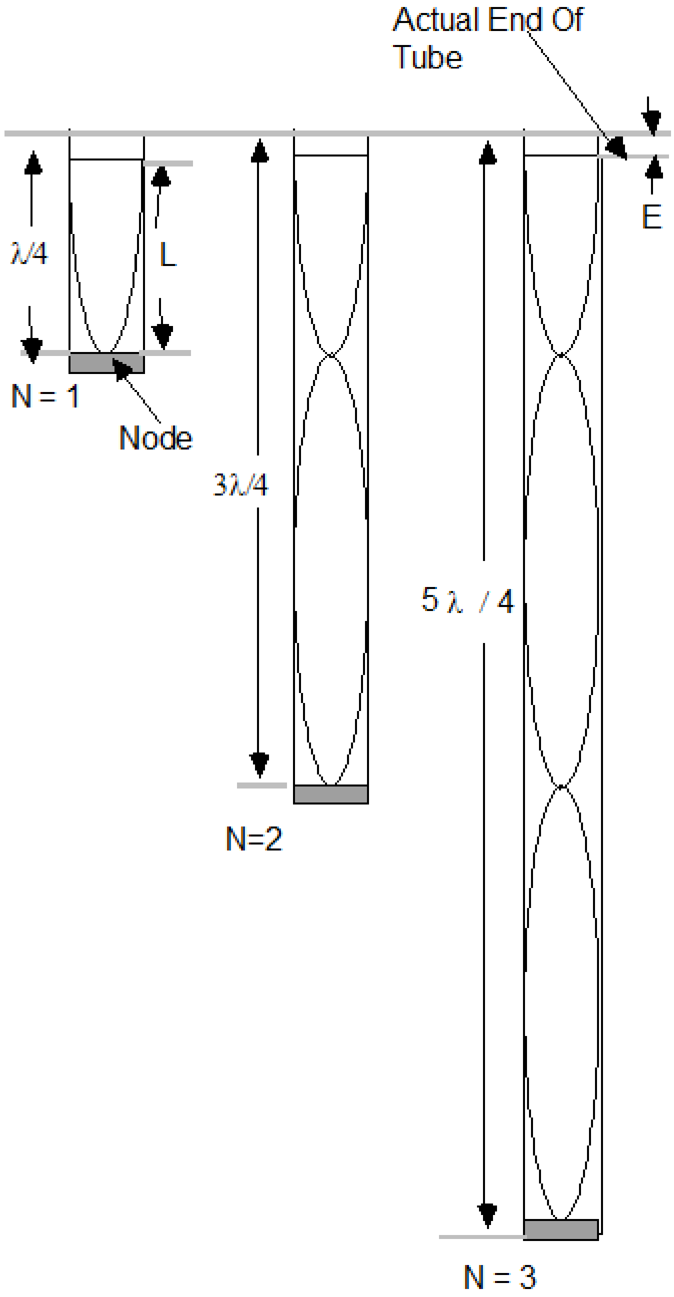


Figure 3.2: Characteristics modes of vibration found as the length of the air column is increased, while driving frequency is fixed. The curves show the distribution of particle displacement by analogy to the appearance of a vibrating string. In the tube, particle displacement is actually parallel to the length. In this experiment, any of these vibrations produces audible sound in the room. Note that the N in the figure above denotes a mode number label. Lower case n is used to indicate the number of quarter wave length in the column.

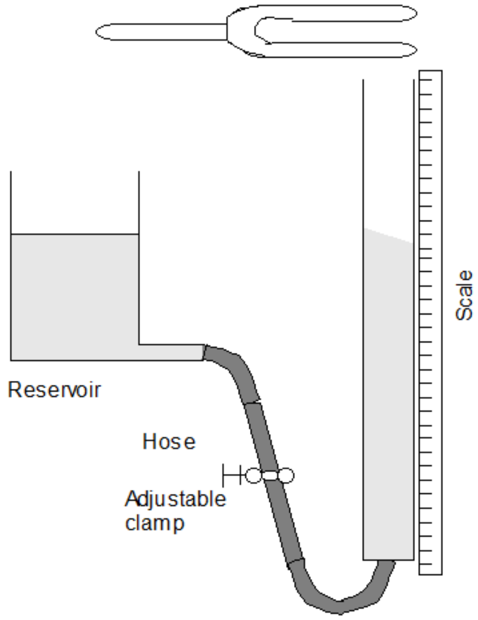


Figure 3.3: The variable-length air column is obtained by regulation of the water level in a vertical glass tube.

We will be able to search for air column resonances by slowly changing the length of the air column. The distance by which the air column is changed in order to find two adjacent resonances is just $\lambda_{\mathrm{air}}/2$. From ν and λ_{air}, we can find C_{air}.

3.3.1 Apparatus and Procedure

We can change the height of the air column by altering the water level, as shown in Fig. 3.3. The tuning fork is put into vibration and held over the tube while the water level is allowed to rise or fall slowly. Resonances are indicated by strong audible vibration of the air in the tube.

The tuning fork should be set into vibration by striking it on the palm of the hand or the heel of your shoe — not by striking a hard surface. The rate of water flow is regulated by clamping the hose; the direction is determined by the reservoir height. The position of each resonance should be determined ($L_1,\ L_3,\ \ldots$) as the average of several trials made with the water falling, and an equal number made with the water rising. The resonances are numbered by how many quarter-wavelengths they encompass, so they are odd-numbered only. This procedure greatly reduces errors caused by the observer's time lag in responding to the audible resonance and reading the position of the moving surface. Put a strip of masking tape on the glass column so you can mark resonance locations with a pencil.

Once the values of L are processed to find $\lambda/2$ we may use Equation (3.1) to find the end correction E from L_1, L_3, and so forth. The E values found from the various L's are then averaged.

3.3.2 Data and Calculations

Since the end correction is difficult to calculate, we find $\lambda/2$ by taking length differences $L_3 - L_1$, $L_5 - L_3$, etc., each of which is theoretically equal to $\lambda/2$. In this different process, what happens to the end correction? If now you average in a simple way the three differences $L_7 - L_5$, $L_5 - L_3$ and $L_3 - L_1$, show that this yields just

$$\lambda/2 = (L_7 - L_1)/3$$

Have you made any use of your measurements L_3 and L_5? This means that you potentially are capable of higher accuracy than this simple averaging method produces. Could you devise a better way of handling the data? G.W. Pierce[1] suggests the following:

$$\lambda/2 = \frac{(L_7 - L_5) + (L_5 - L_3) + (L_3 - L_1) + (L_7 - L_3) + (L_7 - L_1) + (L_5 - L_1)}{10}. \quad (3.3)$$

Why 10?

See if you can show his general result for m total number of differences available and the successive differences $L_7 - L_5$, $L_7 - L_5$, etc., are denoted a_1, a_2, etc.

$$\lambda/2 = \frac{ma_1 + 2(m-1)a_2 + 3(m-2)a_3 + \dots ma_m}{m + 2(m-1) + 3(m-2) + \dots m} \quad (3.4)$$

Use this to compute $\lambda/2$.

As no provision for temperature regulation is made in the experiment, the speed of sound in air must be found at whatever temperature T (in degrees Celsius) happens to exist in the laboratory. To obtain a standardized comparison of results, it is convenient to reduce the value to an equivalent at 0°C, C_0. According to the kinetic theory of gases, C is proportional to the square root of the absolute temperature, so that

$$C_0 = \frac{C}{\sqrt{\frac{273+T}{273}}} = \frac{C}{\sqrt{1 + T/273}}$$

If T is much smaller than 273°C (as will be the case at room temperature), we may express the quantity in the bracket by two terms only of the binomial theorem expansion, giving

$$C_0 = C - \frac{T}{546}V. \quad (3.5)$$

The program of measurements and calculations is shown in the data tables below.

Question 3.7 Speed of Sound in Air:

Frequency of tuning fork $f =$ ____________________ Hz (read from the tuning fork)

Room temperature: $T =$ ____________________ °C

Inside radius of tube: $R =$ ____________________

	Moving up			Moving down			Average	Difference
	Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3		$\Delta_{mn} \equiv L_m - L_n$
L_1								$\Delta_{31} =$ $\Delta_{53} =$
L_3								$\Delta_{75} =$ $\Delta_{73} =$
L_5								$\Delta_{71} =$ $\Delta_{51} =$
L_7								

$\lambda/2 =$ ____________________ (see equation 3.3)

Speed of sound in air, room temperature: $C =$ ____________________

[1]G.W. Pierce, Proceedings of the American Academy of Arts and Sciences, Vol. 60 (October, 1925)

Speed of sound in air, reduced to 0°C: $C_0 =$ ____________________ (see equation 3.5)

Accepted value: 331.7m/s for dry air at 0°C.

Disagreement: ____________________ %

Question 3.8 End Correction:

From average L_1, $E =$ ____________________

From average L_3, $E =$ ____________________

From average L_5, $E =$ ____________________

From average L_7, $E =$ ____________________

$E_{average} =$ ____________________

$E_{average}/R =$ ____________________

3.4 Kundt's Tube Determination of $C_{\textbf{metal bar}}$

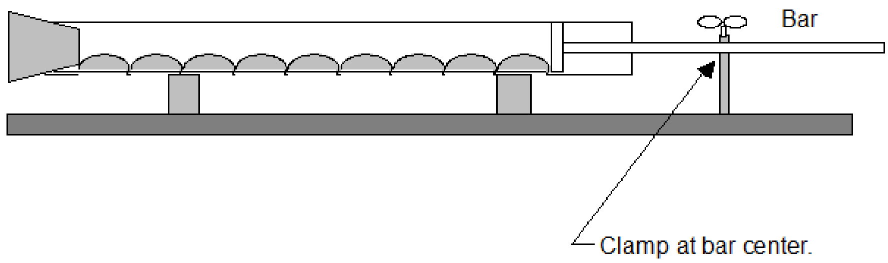


Figure 3.4: Kundt's tube.

3.4.1 Introduction

The tube in this experiment is excited by a metal bar attached to a diaphragm as in Fig. 3.4.

Question 3.9 Is the dust correctly arranged in Fig. 3.4 following excitation of a normal mode?[2] Why or why not?

The metal bar is driven into longitudinal oscillation by stroking it. A steady pull on the bar would be ineffective, however, by putting rosin on a chamois cloth and stroking the bar along its axis, one produces relaxation oscillations in the applied force.

Try it.

This sequence of applied pulses contains frequency components that will effectively set the bar vibrating. You are familiar with relaxation oscillation from your school days when you pushed a long piece of chalk across a blackboard and produced a series of dots. If you need to refresh your memory, take some chalk and

[2] Hint: consider the dust heaps at the closed end.

run it across the board.[3] Since the bar is clamped in the center, it must have a node there and since the ends are free we will have antinodes at the ends.

Question 3.10 How would the lowest mode on the bar look?

We will assume the bar oscillates primarily in its lowest mode.

Question 3.11 Is the bar oscillating transversely or longitudinally?

When the bar is set in motion the gas is also set in motion by the diaphragm attached to the end of the bar.

Question 3.12 What quantity will necessarily be the same in the gas and in the bar? (wavelength, velocity of sound, or frequency?)

When the gas is set into oscillation you will observe that in addition to the expected pattern in the dust, there are present many small ripples or striations. These are produced by the air flowing by the particles . We will not study these striations. For details, see the paper on the bulletin board in the lab room.

3.4.2 Theory and Procedure

Quantitatively we are interested in the dispersion relation for air.

Question 3.13 What is a dispersion relation?

The dispersion relation in air for longitudinal oscillations is $\omega^2 = (\gamma P_A/\rho)(\frac{2\pi}{\lambda})^2$, where γ is the adiabatic constant ($\gamma = c_p/c_v$, c_p and c_v are specific heats at constant pressure and volume), P_A is the equilibrium air pressure in the tube and rho is the density of the air. You will derive this result in the lecture, for now, just accept it as another dispersion relation.

The bar is set into longitudinal vibration by stroking. (When too much rosin has accumulated on the metal bar, it must be cleaned with ethanol.)

Question 3.14 Whenever the bar becomes appreciably heated by friction it must be allowed to cool before attempts are made to set it into vibration. Why?

Question 3.15 Sliding the glass tube within its supports changes what quantity? How? Does this change the patterns in the cork dust?

Ordinarily, the bar will be excited in its fundamental mode for which the length of the bar equals one half of the wavelength for longitudinal waves in the metal, i.e., $\lambda_{\text{bar}} = 2L_{\text{bar}}$.

Question 3.16 How is the wavelength in air related to the distance between dust heaps?

Question 3.17 How is the frequency of the vibrating air related to the frequency of the vibrating bar?

Question 3.18 Use your answers to derive

$$C_{bar}/C_{air} = \lambda_{\text{bar}}/\lambda_{\text{air}}, \tag{3.6}$$

where the longitudinal dispersion relation for the bar is

$$\omega_{\text{bar}} = C_{\text{bar}}k_{\text{bar}} = C_{\text{bar}}\frac{2\pi}{\lambda_{\text{bar}}},$$

and for air:

$$\omega_{\text{air}} = C_{\text{air}}k_{\text{air}} = C_{\text{air}}\frac{2\pi}{\lambda_{\text{air}}}.$$

[3] See E. Rabinowicz, "Stick and Slip", *Scientific American*, 194, 109, May-1956

Remember the C's depend on properties of the medium. Recall that the C's are just the quantities that determine the frequency to be expected for a given spatial disturbance. Since the C's are equal to λf they are also the velocity of sound waves in the medium. We can thus conclude

$$C_{\text{bar}} = L_{\text{bar}} \frac{C_{\text{air}}}{\lambda_{\text{air}}/2} \tag{3.7}$$

From the cork dust we can find λ_{air} and we can measure L_{bar}. Thus we have the ratio of $C_{\text{bar}}/C_{\text{air}}$. Using the C_{air} from the water column experiment we will determine C_{bar}, the speed of sound in the bar. By making the bar out of different materials we could determine C for each material. Then we compare the results with the value from the Handbook of Chemistry and Physics for the metal used. (You can find it on the bulletin of the lab.)

Question 3.19 From your measurements on the Kundt's tube, find the value of C_{bar}. The following table is convenient for this computation.

Kind of metal: ____________________

Length L_{bar} = ____________________

Velocity of sound under room temperature (water column) C_{air} = ____________________

Spacing between dust heaps:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	$a_{Average} = \lambda/2$	λ

C_{bar} (Kundt's Tube)= ____________________

C_{bar} (HC&P)= ____________________

% Error ____________________

Approximately how much faster is sound in metals than in air at STP? ____________________

EXPERIMENT 4

Refraction of Light

4.1 Equipment Required

1. He-Ne Laser
2. 2-45° Prisms
3. Plane Mirror
4. Plastic Block
5. Ruler
6. Protractor
7. Water

Before class, please derive equation (4.4).

4.2 Introduction

Refraction refers to the change in direction of a propagation that occurs as light passes between media having different propagation speeds. Some aspects of refraction relevant to this experiment are described below. Consult textbooks such as *Hecht* or *Born & Wolf* for more extensive discussions.

4.3 Snell's Law

Snell's law, the most basic equation of refraction, is

$$\frac{\sin\theta_i}{\sin\theta_2} = \frac{\text{(speed in medium 1)}}{\text{(speed in medium 2)}} = \frac{v_1}{v_2} \tag{4.1}$$

where v signifies the velocities in the media and the angles are shown in Fig. 4.1. The incident ray, the perpendicular, and the refracted ray lie in a plane. The paths as shown with appropriate changes of the reflected ray, apply also to light traveling in the reverse direction. The index of refraction is defined as $n = c/v$ where c is the speed of light in a vacuum.

Question 4.1 With this definition show that Snell's Law can be written in the more familiar form,

$$n_1 \sin\theta_i = n_2 \sin\theta_2 \tag{4.2}$$

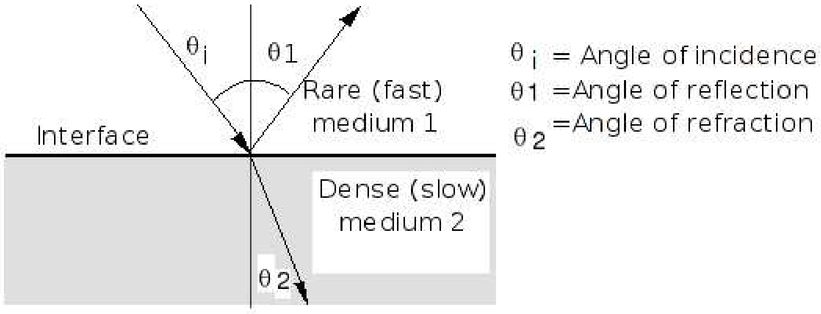


Figure 4.1: Refraction at an interface between two different media

The ray as used here in geometrical optics amounts to a construction line indicating direction of travel, but it has no direct physical significance. We can observe only the behavior of a pencil of light of some small cross section, sometimes described as a "bundle of rays," where each ray in the bundle is thought of as traveling parallel to the ray actually drawn in a diagram such as Fig. 4.1. The very small angular spread of the beam from the small laboratory laser insures that this beam is not visibly different from Fig. 4.1.

4.4 Refraction through plastic

Shine the laser beam through the plastic block provided; note the refraction entering and leaving, compare direction of the incident ray with the direction of the exiting ray. Has the ray's direction changed upon passing through the block?

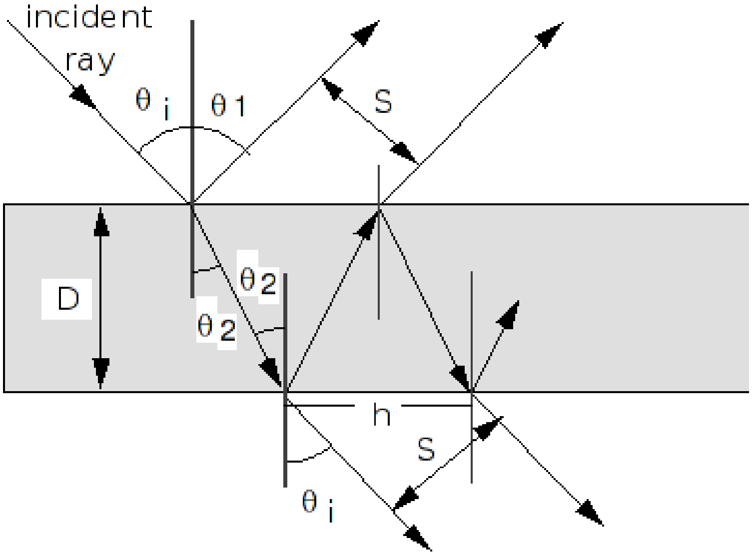


Figure 4.2: Multiple refraction

Figure 4.2 shows the behavior of light striking a parallel-sided block of plastic. In this figure we also show reflected rays which are always present to some extent at the interfaces. The angle of reflection always equals the angle of incidence. By using this principle, plus the forward and reverse forms of Fig. 4.1, one can show that several (successively weaker) rays leave the plastic on each side.

Question 4.2 Why are the rays getting weaker?

The geometry of the problem makes it clear that

$$h = \frac{s}{\cos\theta_i}. \tag{4.3}$$

If you take the system and apply Snell's law, you can show that the spacing between successive rays, s, is determined by

$$s = \frac{2D\sin\theta_i\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}} \tag{4.4}$$

Question 4.3 Combine equations (4.3) and (4.4) to find n in terms of the block thickness, angle of incidence, and ray spacing, h.

Question 4.4 How can you determine that the laser beam is normally incident on the surface?

4.5 Determination of index of refraction by multiple refraction

Set the plexiglass block and laser up to cause multiple refraction, then make the necessary measurements to use your derived result to question 4.3 and calculate the index of refraction for plexiglass.

Question 4.5 Use the method of your choice to find the index of refraction for the plexiglass. What is the n, and what does this n tell us about the velocity of light in plastic compared to light's velocity in air?

4.6 Total internal reflection

If you have not tried already, aim the laser through the slanted side of the plexiglass block. Observe the behavior of the laser ray that leaves the plexiglass as you vary the internal angle of incidence on the plexiglass-air surface.

Question 4.6 Find the critical angle of incidence where light does not refract out of the plexiglass. At what angle does Snell's Law predict this? What happens to the light?

4.6.1 Uses for total internal reflection

Total internal reflection is a useful phenomenon which is absolutely essential for fiber optics. It also presents other capabilities which you will shortly investigate

Question 4.7 If you were tasked with efficiently reflecting light, come up with two ways of doing so.

Question 4.8 Compute the critical angle for crown glass, which has an n of 1.515. If light is incident on a 45° prism, as in Fig. 4.4, what will happen to it?

Question 4.9 If we have two parallel rays R_{Top} and R_{Bottom} incident on the prism as shown, what will be their relative positions upon leaving the prism? What could you do with two such prisms?

Arrange the prisms as shown in figure 4.5 and shine the laser through normal to the first prism. Because the prisms are not optically smooth, the beam will not continue straight through the system.

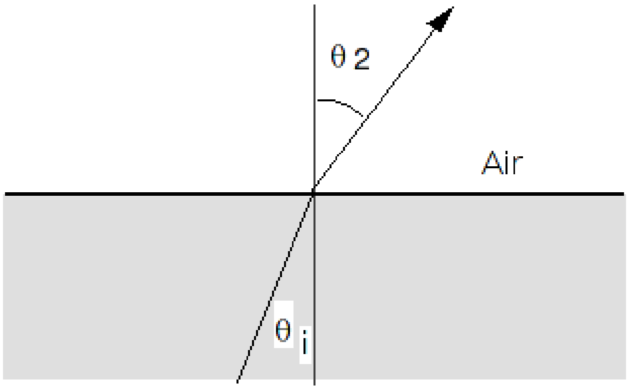


Figure 4.3: Light refracted from plexiglass to air

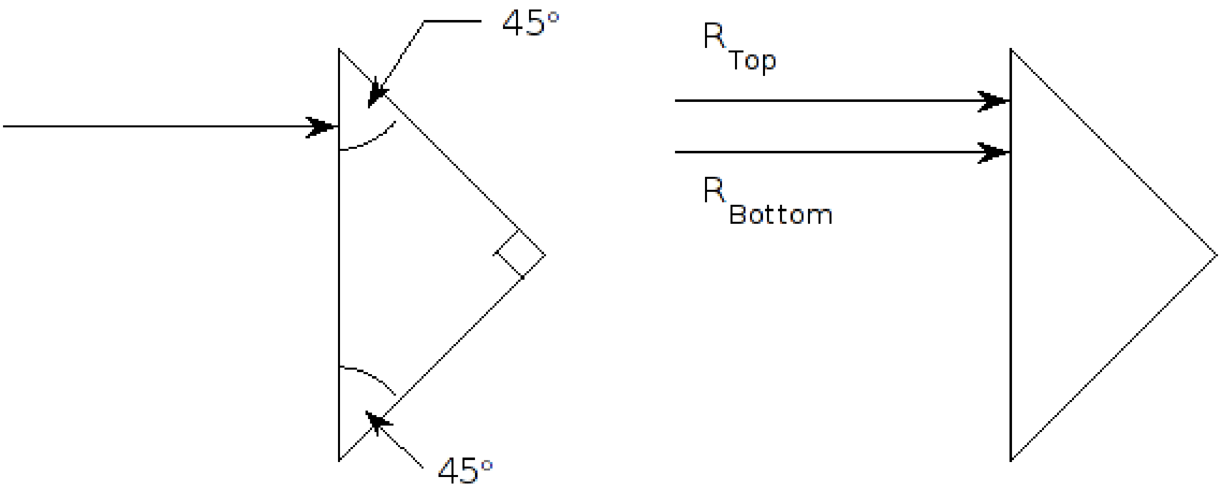


Figure 4.4: Light incident on a 45° prism

Question 4.10 If you add water in between the prisms, what happens? Why?

4.7 Images and the Method of Parallax

Refraction's property of bending light rays manifests in how we see images. After light has passed through a refractive material, it can change its apparent position, an important consideration for fish harpooners and physicists alike.

Place the block of plastic over an object, such as this lab book. Where does the object now appear to be located when viewed from above? This point can be determined theoretically through Snell's Law and experimentally through the method of parallax.

Question 4.11 Use Snell's law to derive an expression for the location of P', as shown in Fig. 4.6. It will be much easier if you consider rays where Φ_I and Φ_R are small and you can use the approximation that $\tan\theta \approxeq \sin\theta$. You should get an expression for s' in terms of indices of refraction and s.

Question 4.12 How would your expression change if you selected a different ray from P than the one shown? Can you conclude something about where the rays leaving from P appear to come from?

4.7.1 Using Parallax

First let us decide what type of image we see in a plane mirror. Obtain a mirror and place a pencil in front of it.

Question 4.13 Is the pencil's image virtual or real?

In the later labs, you will frequently use the method of parallax to find the properties of various optics. The method is thus: Use a pencil as your object in front of the mirror; place a second pencil behind the mirror such that you can see the image and the second pencil sticking up above the edge of the mirror. Move your head side-to-side, is there relative motion between the image and the pencil?

If there is, then your second pencil is not at the same position as the image. Move the pencil until it is in precisely the same place as the image of the second pencil. This is the method of parallax, in a nutshell.

Question 4.14 Use the method of parallax to find the image distance for both the block of plastic and the mirror. Compare these with the calculated positions.

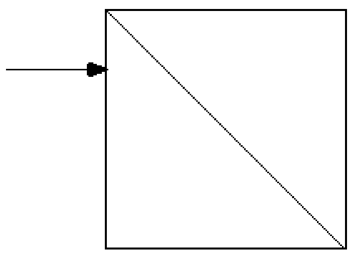


Figure 4.5: Experimental setup for two prisms

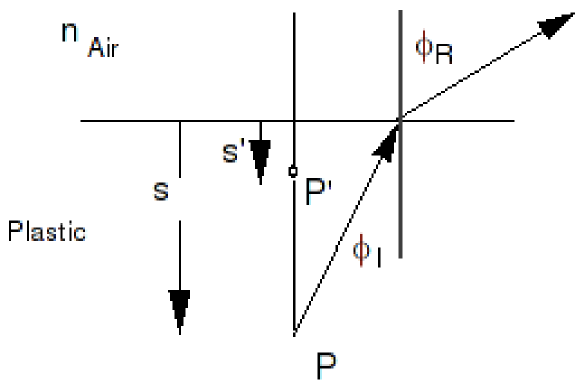


Figure 4.6: The object at P is far removed from its image at P'.

EXPERIMENT 5

Thin Lenses

5.1 Equipment Required

1. Laboratory Jack
2. He-Ne Laser
3. Large Converging and Diverging Lenses
4. Card-stock Screen
5. Optical Bench
6. Point Light Source
7. Card-stock masks
8. Protractor and Compass
9. Sphereometer
10. Lens Mounts

5.2 Introduction

Light is (obviously) an extremely important part of science and life. You already know about its importance in life, and this experiment aims to teach you about its use in science. Thin lenses are the most simple optical devices you encounter in the lab, and in this experiment you will be introduced to their properties and also some of the limitations of optical systems employing lenses.

5.3 Initial Observations

From the last experiment, you know that light is refracted (bent) upon entering a medium with a different phase velocity. You also know how lenses are generally shaped.

Question 5.1 Light is refracted unless it hits what spot on the lens?

Question 5.2 Draw a simple double-convex lens and show that most light rays are bent by the lens.

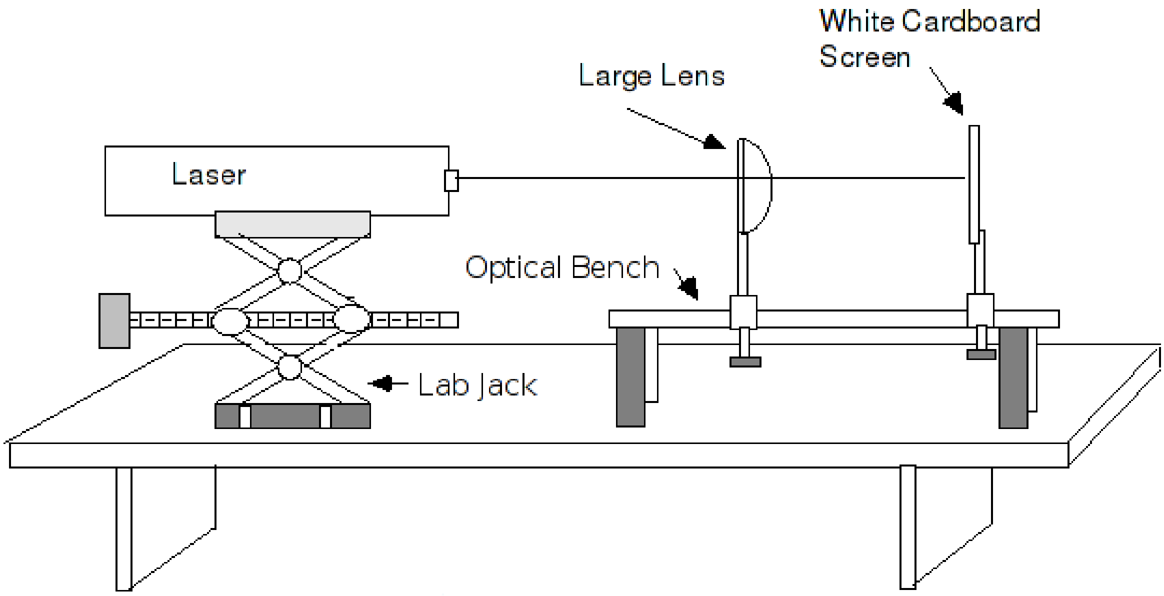


Figure 5.1: First experiment's setup

5.4 Focus for a Plano-Convex Lens

Let us find where horizontal rays cross the axis of the lens (the optical axis). Set up an optical bench as in Fig. 5.1.

Align the system such that the laser spot on the screen does not move much as the screen is moved the full length of the optical bench. Mark the spot on your screen. Make sure the laser beam passes through the lens axis.

Move the screen very near the lens and observe the spot on the screen as the laser beam is moved from the axis of the lens to the edge without changing the angle of the laser. Note the direction of spot movement. Now move the screen far away and repeat the steps above.

Find a position for the screen where there is no motion of the spot as the horizontal laser beam moves from the center to edge. You have found the focal point, which is the intersection point of the screen and the optical axis of the lens. The distance from the center of the lens to the screen is coincidentally called the focal length.

Question 5.3 Measure the focal length for the lens with the plane side facing the laser, then turn the lens around and measure it again.

Now, there are many rays entering a lens which are *not* parallel to the lens axis. Try tilting the lens from right to left.

Question 5.4 Repeat the experiment above and describe what you observed. How did the determined focal length compare with that for the horizontal rays?

Question 5.5 What happens to a ray which first passes through the focal point then strikes the lens as shown in Fig. 5.2? Check your answer with the laser.

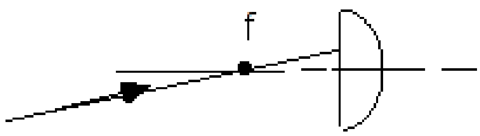


Figure 5.2: The laser can be positioned to go through the focus.

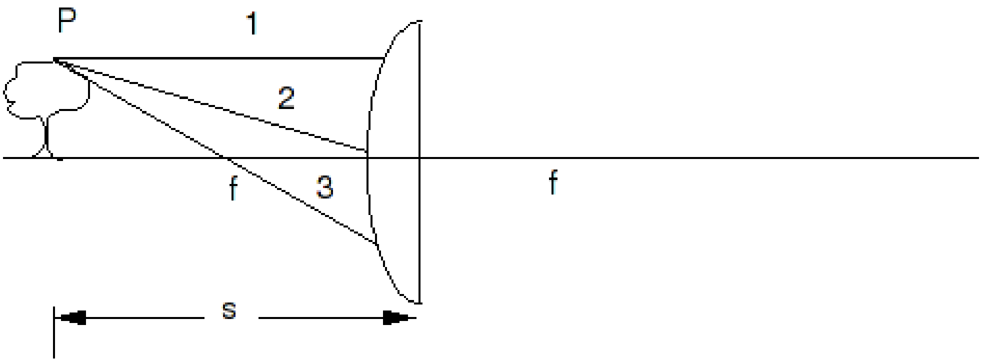


Figure 5.3: Rays from a tree going through a plano-convex lens

5.5 Images through thin lenses

To reach the full potential for a lens, we must be able to account for light which is not parallel.

5.5.1 Theory as demonstrated by a tree

Imagine light from a source at some distance S from a lens as in Fig. 5.3. Light rays reflected from the tree at point P diverge in all directions. Let us consider the rays labeled 1,2, and 3.

Question 5.6 Copy Fig. 5.3 into your lab notebook and draw the continuations of rays 1,2, and 3. Do the rays converge at any point?

Question 5.7 If you were to place the screen an arbitrary distance behind the lens, what would you observe about the light from point P?

Question 5.8 Consider question 5.6. If they rays did converge, then what would be at that point?

5.5.2 Images from Incandescent light sources

Place an incandescent light source on the bench well outside the focal length of the lens. Move the screen until you can see a focused image.

Question 5.9 The distance of the screen from the lens is called the image distance, record this distance. Is the image inverted or erect?

Question 5.10 As you move the lens toward the light source, what happens to the magnification? How about away from the light source?

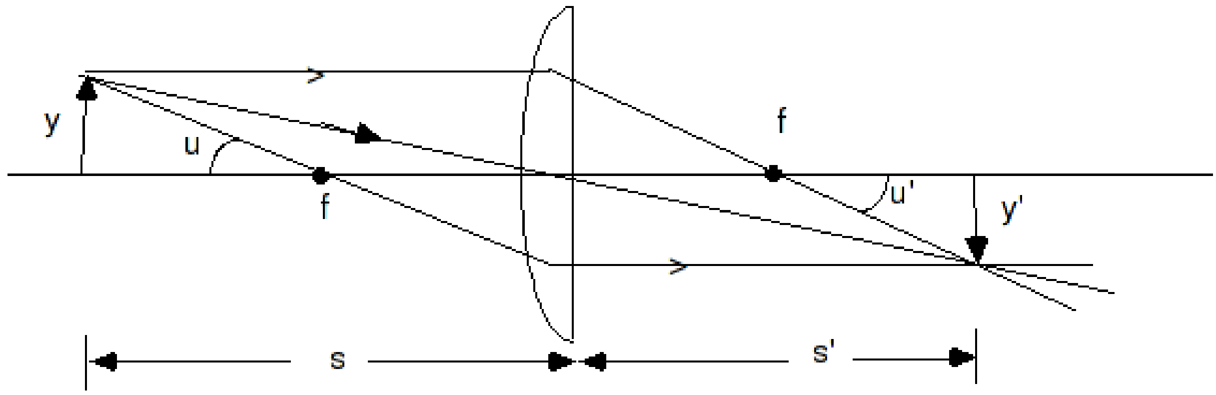


Figure 5.4: An object at y outside the focus f of a converging lens produces an image at y'.

Now move the lens so that the light source is inside the focal length. Try finding the image, first with the screen, then second with your eye.

Question 5.11 Measure the location of the image. Is the image erect or inverted? Is it real or virtual?

5.6 Thin Lens Theory

Using the experimental results you have, let's see if we can relate object and image distance for a lens. The three rays from the tip of the arrow pass through the lens as shown.

From Fig. 5.4, you can find two triangles which lead to the equations

$$\tan u = \frac{y'}{f} = \frac{y}{s-f} \tag{5.1}$$

and

$$\tan u' = \frac{y}{f} = \frac{y'}{s'-f}. \tag{5.2}$$

Question 5.12 Use Fig. 5.4 and the two earlier equations to get the thin lens equation,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{5.3}$$

Question 5.13 Find an expression for the magnification in terms the object and image distances. Remember that magnification can be positive or negative, positive if the image is erect, negative otherwise.

5.6.1 Experimental validation of thin lens equation

You have already taken measurements for the locations of the light source and image it creates going through a thin lens. Now, you will see whether the thin lens equation holds.

Question 5.14 Check your previous results to see how accurate the thin lens equation is. Is it possible to have a negative s'? What would this mean?

If an object is very far away such that the rays from a point on the object are essentially parallel when reaching the lens, they will converge at the focus of the lens. This will leave an image of the object which appears at the lens' focus. You can use this to approximate the thin lens equation and find the location of the focus of a lens without needing to know the distance to the object.

Question 5.15 Take the thin lens formula and make an appropriate approximation for a distant object. Using a lamp at the other end of the room, find the focus of the lens and compare this result to your others. How do they compare?

End of Part I —Stop here

Beginning of Part II

5.7 Diverging lens

The concave lens has different properties than a converging one, which you will investigate. Return to the setup from Fig. 5.1 with the diverging lens in place of the converging one.

Question 5.16 Use the screen to follow the laser as it translates across the lens. Make a ray diagram of what you see.

5.8 Lens aberrations

Return to the plano-convex lens. Quickly find the focal length using whichever method you are most comfortable with. (You might have a different lens than last class.) Set the screen to the focal length and translate the laser to an extreme edge of the lens.

Question 5.17 Which direction did the laser point move? What does this indicate about the focal length for rays incident on the edge of the lens compared to the axis of the lens?

The fact that rays coming through different parts of the lens cross the axis at different points means that this lens has an aberration. Specifically, this is a spherical aberration. It is more pronounced in large lenses. A parabolic lens does not exhibit spherical aberrations, but it is more difficult and costly to construct.

Remove the laser and switch to the incandescent light source with an arrow mask. Focus the arrow on the screen as clearly as you can.

Question 5.18 If you place a mask over the bottom half of the lens, what will happen? Be sure of your answer before you try it.

Now use a mask to cover the center portion of the lens.

Question 5.19 Find the new focal length for the masked lens.

Use masks to block off the edges of the lens. Use different aperture sizes and find the focus for each aperture.

Question 5.20 Are there changes in brightness and contrast between apertures? Which aperture has the best brightness? Best contrast?

5.8.1 Aberration theory

The envelope of rays refracted by a lens is called the caustic. The intersection of the marginal rays and the caustic is called Σ_{LC}, the circle of least confusion. At this point, there is minimal image blur. An illustration is provided in Fig. 5.5.

Question 5.21 What happens to the position of Σ_{LC} when the aperture size is decreased? Does this force you to refocus the image?

Question 5.22 Measure the spherical aberration for the plane side facing the light source and facing the screen. Which configuration exhibits less aberration?

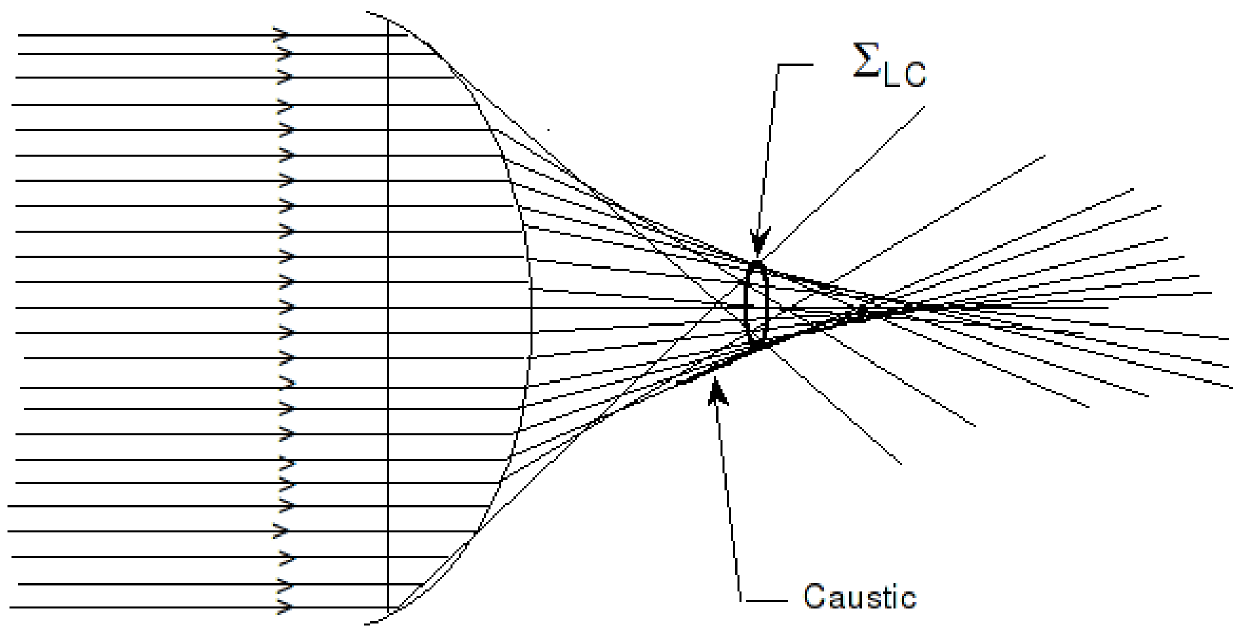


Figure 5.5: A demonstration of the rays for a converging lens, showing the caustic and circle of least confusion

5.9 The Lens Maker's Equation

Consider a light ray from the point source s approaching a spherical surface of radius R_1 and index of refraction n_1, as illustrated in Fig. 5.6. The relations evident are

$$\begin{aligned}\theta_I &= u + \psi \\ \psi &= \theta_R + u' \\ n_0 \sin\theta_I &= n_1 \sin\theta_R\end{aligned}$$

where n_0 is the index of refraction for air, $\cong 1$.

For small angles, the last equation becomes

$$n_0\theta_I = n_1\theta_R \qquad \tan u = \frac{y}{s+a}$$

$$\tan u' = \frac{y}{s'-a} \qquad \tan\psi = \frac{y}{R-a}$$

where $a \ll s$ and $a \ll s'$, then $\tan\theta \cong \theta$. At this point, you can combine all of the equations and you are left with

$$\frac{y}{s} + \frac{y}{R} = \theta_I = \frac{n_1}{n_0}\theta_R. \tag{5.4}$$

This can be rewritten in a way similar to the thin lens equation through

$$\begin{aligned}\theta_R + \frac{y}{s'} &= \frac{y}{R} \\ \frac{y}{s} + \frac{y}{R} &= \frac{n_1}{n_0}\left(\frac{y}{R} - \frac{y}{s'}\right) \\ \frac{n_0}{s} + \frac{n_1}{s'} &= \frac{n_1 - n_0}{R}\end{aligned} \tag{5.5}$$

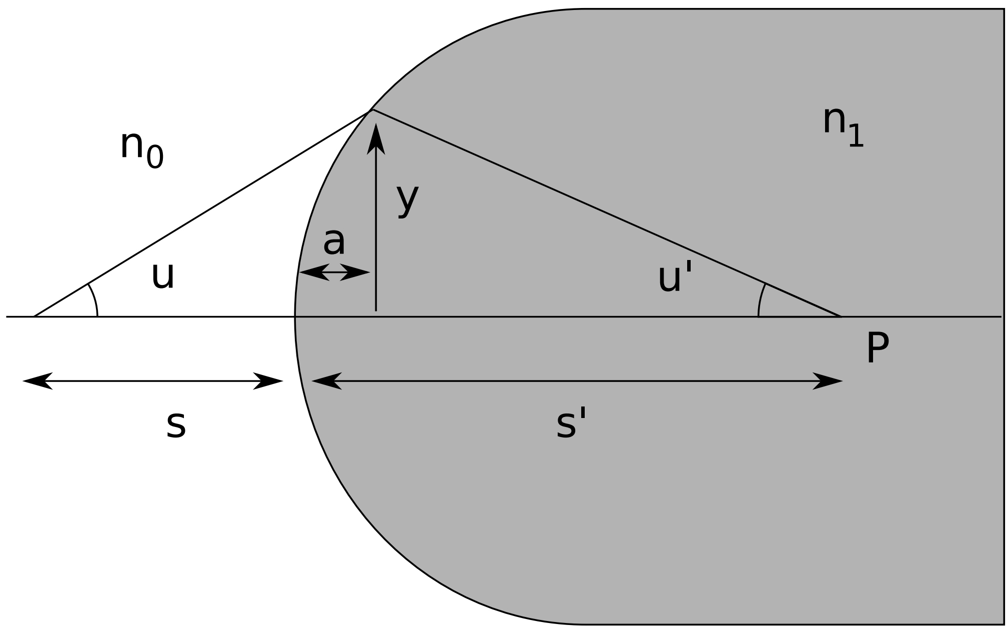


Figure 5.6: A ray incident on the side of a lens

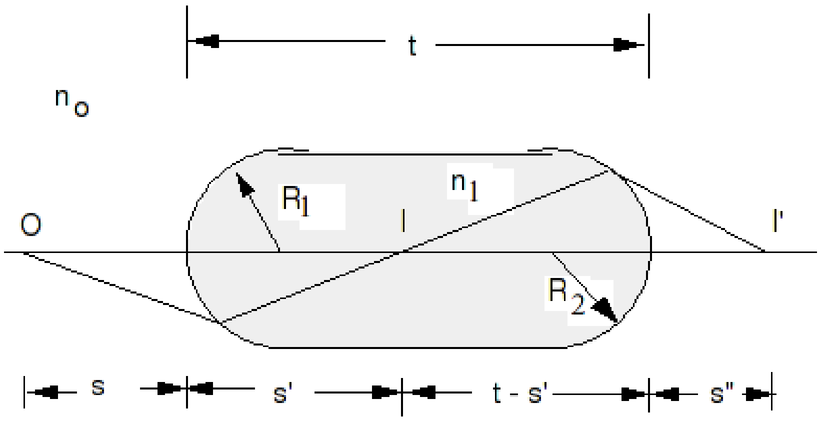


Figure 5.7: Light from the object at O, after hitting the first surface of the lens appear to come from an object at I, and after hitting the second surface reaches an image location of I'.

At this point, we have a relationship between the object and the image due to light hitting the lens' first surface. The only further need is to repeat the procedure for the second surface of the lens. The picture will look like Fig. 5.7

After the second surface, the image location s'' is described by

$$\frac{n_1}{t - s'} + \frac{n_0}{s''} = \frac{n_0 - n_1}{R_2} \tag{5.6}$$

If you assume that the lens is very thin, then t is small and $n_1/(t - s') \cong -n_1/s'$. Then the equations for the first and second surfaces can be combined, leading to

$$\frac{n_0}{s} + \frac{n_0}{s''} = (n_1 - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{5.7}$$

For the second surface R_2 is usually negative. Now, recall that for a thin lens in air ($n = 1$), the equation relating the object and image locations is

$$\frac{1}{s} + \frac{1}{s''} = \frac{1}{f}$$

Question 5.23 Determine an expression for f (or $1/f$) in terms of parameters characterizing the lens.

Question 5.24 Develop an equation from Fig. 5.8 for R. Using the spherometer, measure R for the lens.

Question 5.25 Use the radius of the lens and the index of refraction for glass ($n = 1.5$) to determine the focal length for the lens in air. Compare this result with the experimentally determined value.

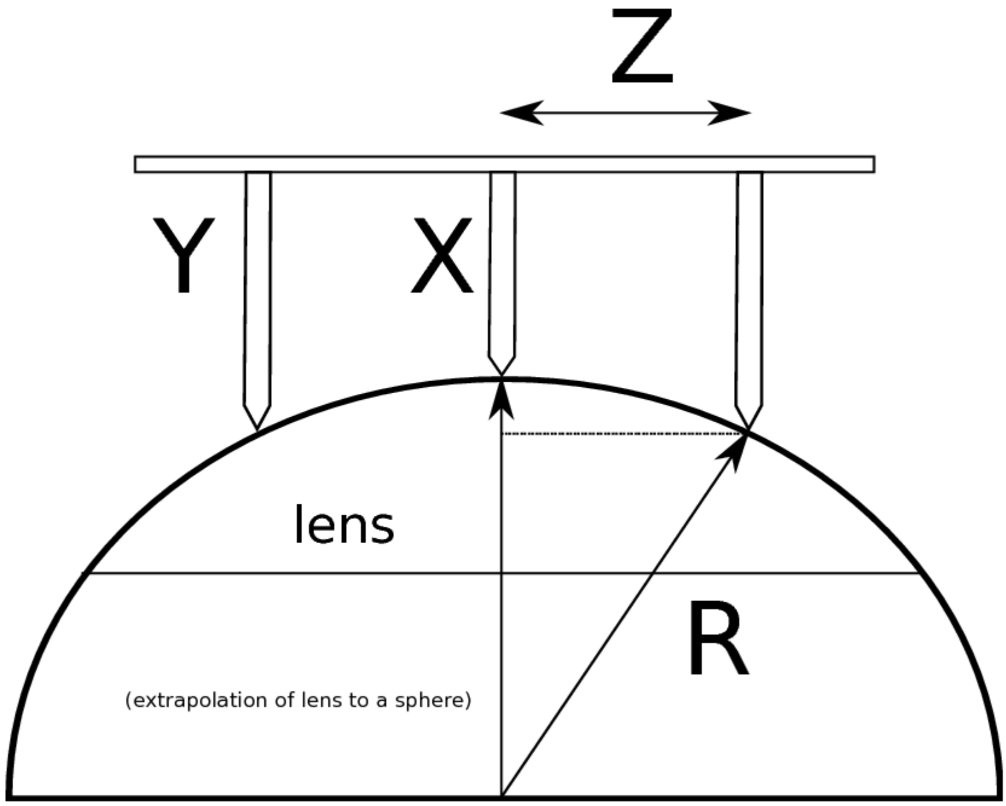


Figure 5.8: A spherometer can be used to find the radius of a lens, with a little geometry.

EXPERIMENT 6

Polarization of Light

6.1 Equipment Required

1. 3 linear polarizers
2. glass slab
3. He-Ne laser
4. 2 quarter-wave plates
5. half-wave plate
6. optical bench
7. lab jack
8. beaker
9. cane sugar

6.2 Introduction: What is polarization?

Polarization is the manner in which the electric field points; it is perpendicular to the direction in which the light propagates. Consider a light wave propagating into the paper, in the $\hat{z}$ direction. Then the electric field vector **E** would point be in the xy plane, along the paper. This is illustrated in Fig. 6.1.

For unpolarized light sources, such as incandescent lamps, the direction of the **E** vector moves around in the plane of polarization randomly with time. The average strength of the polarization in the x and y directions are the same over time. If we consider the propagation of a ray of light in space, then we see that the **E** vector resides entirely in a plane. This light is termed linear or plane polarized.

Even in the case of a linear polarized wave, the **E** vector at a given value of z varies with time, often oscillating. In the case of light from a laser source, this variation with time is sinusoidal.

Question 6.1 Draw the **E** vector with respect to time if it is laser light linearly polarized in the vertical direction.

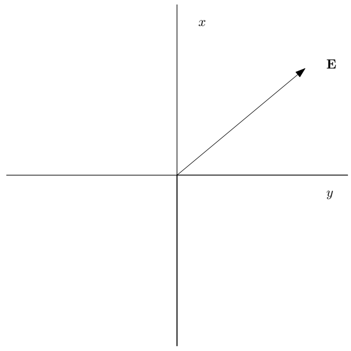


Figure 6.1: The **E** vector at an instant in time

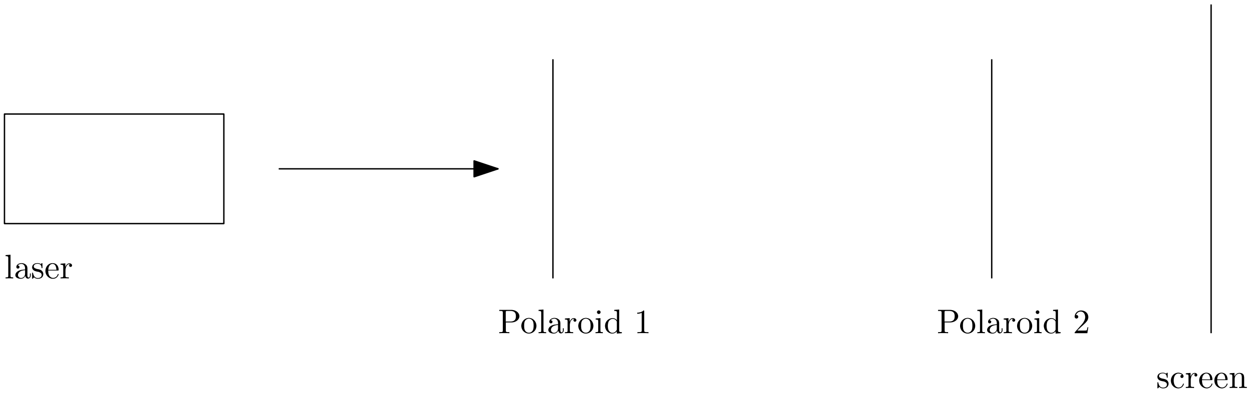


Figure 6.2: The primary laboratory setup in this experiment

6.3 Linear Polarization With Polaroids

In practice, the easiest way of getting polarized light is taking initially unpolarized light and processing it to get rid of light polarized in an unwanted direction. This is done with polaroids.

The basic structure of a polaroid is a series of parallel metal strips, spaced apart by much less than the wavelength of light. These metal strips selectively absorb light parallel to the strips, only allowing light perpendicular to them to continue propagating through the polaroid. With optical light, it is not practical to use metal strips, and iodine molecules oriented through stretching of the material serve the same purpose.

When plane polarized radiation falls on a polarizer at an arbitrary angle, the component along the polarization axis is transmitted, but the component along the perpendicular axis is absorbed.

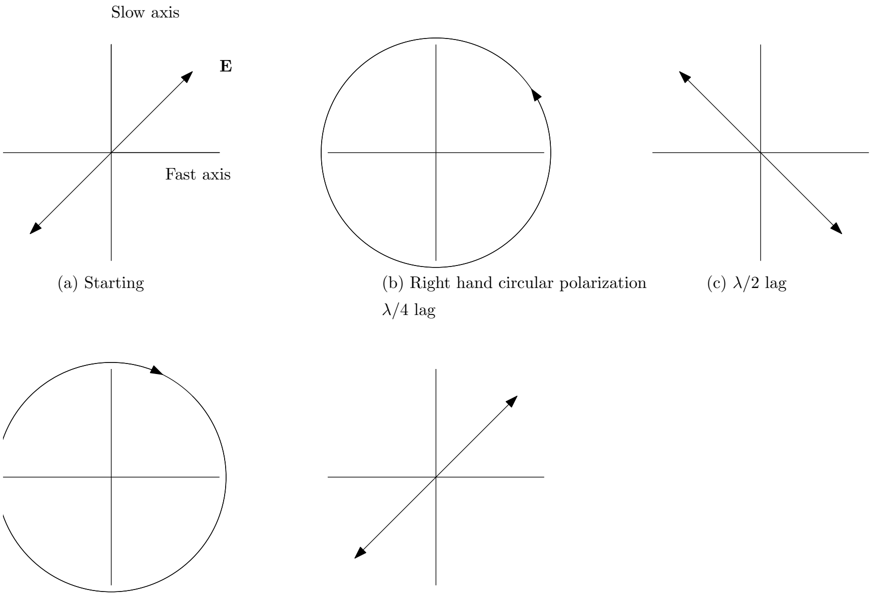
Question 6.2 In figure 6.1, we see the **E** vector's direction. In what direction would this vector point after the light passed through a vertical linear polarizer?

Question 6.3 What determines the direction of polarization for light which has passed through a sequence of polarizers?

Since the intensity of an electromagnetic wave is proportional to the square of the amplitude of the **E** field (it is a wave, after all), the intensity of previously polarized light is proportional to $\cos^2 \theta$.

Set the laser and optical bench such as in Fig. 6.2.

Question 6.4 Vary the position of the second polarizer and measure the light intensity of the resulting beam as a function of the angle.



(a) Starting

(b) Right hand circular polarization $\lambda/4$ lag

(c) $\lambda/2$ lag

(d) Left hand circular polarization $3\lambda/4$ lag

(e) Full λ lag

Figure 6.3: The progression of polarizations for linear polarized light traveling through a birefringent material

6.4 Circular Polarization

Plane polarized light of a single frequency can be converted into circularly polarized light by means of a birefringent material. A birefringent material has different indices of refraction for light polarized in different directions. This means that the phase velocity for light polarized in the two directions is different, and they become out of phase. This makes elliptically polarized light.

The two axes are often called the fast and slow axes. A special case of elliptically polarized light occurs when the **E** vector spins in a circle. This is imaginatively called circularly polarized light.

In order to create circularly polarized light, you start with linearly polarized light which is evenly split between the fast and slow axes. As the light propagates through the birefringent material, the light polarized in the slow axis lags compared to the light polarized in the fast axis, until it is lagging by a quarter wavelength. At this point, the **E** vector is spinning in a circle, and you have circularly polarized light. An optical element which does this for linearly polarized light is called a quarter-wave ($\lambda/4$) plate.

If the light goes through birefringent material twice as far as this, it switches back to linearly polarized light in the perpendicular direction. This is because the lag between light going through the slow and fast axes is $\lambda/2$, which gives the optical element which does this its name, the half-wave plate.

6.5 Polarization through Reflection

When light encounters an interface between two materials of different indices of refraction, then light of different polarization directions behave differently. At a polarizing angle, called *Brewster's angle*, the reflected light is all polarized in one direction. This happens when the reflected and refracted light are perpendicular to each other.

Question 6.5 Using Snell's law, determine Brewster's angle for light from air incident on an object with an index of refraction n.

Now, set up a laser and reflect its light off of the glass slab. You can use the polaroid to see whether the reflected light is polarized.

Question 6.6 Experimentally determine Brewster's angle for the glass slab. Compare this to the theoretical predictions, assuming that the glass slab has an index of refraction $n = 1.51$.

6.6 Experimenting with circular polarizers

Set the experiment up in this order: laser, polaroid, $\lambda/4$ plate rotated 45°, $\lambda/4$ plate, polaroid, and screen. Keep the second $\lambda/4$ plate rotated 45° from the second polaroid.

Question 6.7 What sort of behavior do you expect as you rotate the second $\lambda/4$ plate-polarizer pair? Perform the experiment. Can you tell whether the light is right-hand or left-hand circular?

Question 6.8 Put a half-wave plate in between the quarter-wave plates. At what angle for the second plate-polarizer pair now allow light to pass?

6.7 Rotation of the Plane of Polarization by Optically Active Substances

Certain sugar solutions and turpentine are common substances which are capable of rotating the polarization of light which passes through them. For a standard length of optical path through the solution, the degree of rotation caused by the passage of linearly polarized light through a sugar solution is a measure of the concentration of the solution.

Place an empty beaker between the linear polarizers and cross them to extinguish the light.

Question 6.9 Fill the beaker with water. Does this change the polarization of the light going through the beaker? What happens when you add sugar to the water and mix it?

Question 6.10 What is the best method for measuring the change in polarization? Using this method, see the change when the sugary water mixture is at saturation. Is the polarization change strongly, weakly, or not dependent?

6.8 Polarization by Scattering

If you have an opportunity, take your linear polarizer and observe different portions of the sky. What regions are there where scattered sunlight is more polarized?

Some insects and marine life can detect the polarization of light.

Question 6.11 What purpose does this serve for the animals?

Light beneath the surface of the ocean, or even a pond is polarized to a certain degree. The polarization is predominantly horizontal, with a strength from 5% to as much as 30%. The water flea Daphnia tends to swim in a direction perpendicular to the light polarization, though scientists (or at least physicists) do not know why. The horseshoe crab is presumed to navigate with respect to the polarization direction.

EXPERIMENT 7

Interference And Diffraction

7.1 Equipment Required

1. Laser
2. Optical Bench
3. Showcase Lamp
4. Screen (White Card)
5. Micrometer
6. Rubber Bands
7. Red and Blue Filters
8. Slit Film
9. Two Clean Microscope Slides
10. Ball Bearing

Very Important: The material in the initial pages is a Pre-Lab. You are expected to come to lab with your answers to these questions completed and ready to turn in to the TA.

7.2 Interference

As children, we all tossed stones in a pond or lake and observed the ripples coming from the point where the stone entered the water. We may have concluded that the ripples were caused by the water being displaced up above the surface by the stone which now is below the surface. We might further have concluded that the ripples were just this water sliding along the surface.

Further examination of a fishing cork as such a disturbance goes by reveals that the cork does not travel along with the ring but simply bobs up and down. This means our conclusion above is incorrect. What propagates through the water is the wave, *not the water*.

The wave propagates by exerting a vertical force on a section of water in front of it, while gravity and surface tension act as restoring forces. The section of water is restored to its original position by transmitting its energy to a neighbor farther along.

The movement of water up and down and back and forth makes the cork bob. Now most of us were not content with one rock at a time, soon we started throwing handfuls and we got rather complicated patterns

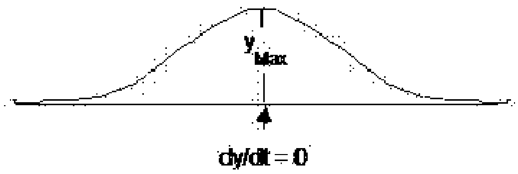


Figure 7.1: Profile for a ripple

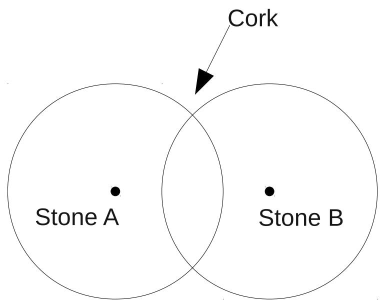


Figure 7.2: Two stones are dropped into the water at the same time. The circles represent the leading edge of the ripples.

of overlapping rings. Instead of pursuing what appears to be a difficult problem, let's instead look at the pattern from just two stones as shown in Fig. 7.2.

A wave emanates from each stone. If we place a detector (such as a cork) in the water we would not be surprised to find that when the two rings intersect at the cork the amplitude (distance it bobs above the undisturbed surface) is twice that of a cork meeting only one ring.

Behind the ring of water there may be a valley. We next might ask what do we do when the cork has a valley and a ring (or maximum) present simultaneously? Observation shows that if "the valley is as high as the mountain" the cork is not displaced. This shows that we should add algebraically the amplitudes of the waves: $A_{Total} = A_{max} + A_{valley}$ but $A_{valley} = -A_{max}$ thus $A_{Total} = 0$. Again we are not surprised by this result since as the maximum approaches it, it is trying to move the cork up and the approaching valley is attempting to move it down, the net result is cancellation.

The important result we must remember is that we algebraically add the amplitudes.

Let us now replace our two stones A and B in Fig. 7.2 with sticks A and B which are bobbing up and down at the same frequency in a regular manner in the water. This means they will launch many valleys and hills (minima and maxima), all with the same distance between adjacent minima or maxima.

Question 7.1 What is this distance called?

If we assume the waves leaving the bobs are not damped very much and that A and B are close together when compared to the observation distance r, then we can describe them with the expression where A is approximately constant.

$$y = A\cos\left(\frac{2\pi}{\lambda}r - \omega t\right) \tag{7.1}$$

where y is the displacement of the water from the undisturbed level, r is the distance from a stick to observation point. ω is the angular frequency of stick and t is the time.

Question 7.2 Can A really be independent of r for a real system? Support your answer.

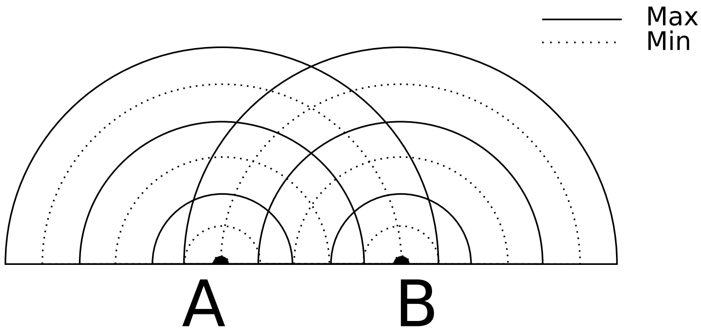


Figure 7.3: Wave pattern for two sources

Question 7.3 Can you determine where you would place a cork such that it would never bob up and down?

Question 7.4 Do so on the diagram and reproduce it in your notebook.

Question 7.5 Find as many places as you can and indicate on the drawing.

Question 7.6 Where would you place the cork in order to have the maximum bobbing amplitude?

Question 7.7 Locate all possible points, also indicate on the drawing.

The phenomena discussed above is called interference and is a property of waves. You could go through the same step for two loud speakers emitting sound waves and again you would find that the amplitudes must be added algebraically. Also you would find points of maximum signal and points of minimum signal.

Elastic waves in solids also exhibit interference. This is a general property of waves.

An obvious next question is "Does light behave as these other phenomena and hence have a wave character?" Alternatively we might ask "Does light exhibit interference?"

Before attacking this question let us pause for a moment and discuss the flow of energy from our sources to the detectors. Let us first take our cork and attach a long thin needle to the top of it and then attach a sandpaper sail to the needle. A second piece of sandpaper is suspended from an overhead support and placed in contact with the sail. As the cork bobs it will cause the sandpapers to rub each other and heat up their surfaces.

Question 7.8 What is the source of this heat energy?

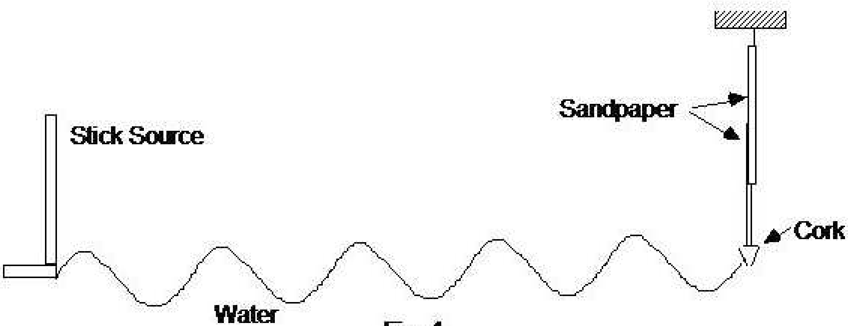


Fig. 4

Now consider the case of Stick A oscillating and Stick B at rest. The cork will clearly oscillate and there is a flow of energy to the cork. Turn Stick A off and turn Stick B on. Again there will be a flow of energy from Stick B. Now place the cork at one of the points of total destructive interference located earlier, call it point C.

Question 7.9 Now with A and B oscillating how much energy flows to the cork at point C? If you turn A off what happens?

Question 7.10 Does turning A or B off have any effect on the energy emitted by the other? Support your answer.

In the lecture you will learn that the power flow in a wave is proportional to the square of the amplitude, i.e. for $y = y_o \cos(kr - \omega t)$ the power flow (S) is $S_{\frac{joules}{m^2 \cdot s}} \propto y_o^2$

Question 7.11 If two waves are present then you might be tempted to write $S_{Total} \propto (y_A^2 + y_B^2)$. Discuss evidence that this is incorrect.

Question 7.12 Is the expression $S_{Total} \propto (y_A + y_B)^2$ consistent with our previous discussions?

Remember that in order to compute power flow, you must add then square. Do not square then add.

Question 7.13 Discuss why you do or do not think this method of computing energy flow is reasonable for the cork in the water.

7.2.1 Light

We next will try to answer our previously posed questions about light. You start by getting two sources of light (such as two flashlights) analogous to our Sticks A and B bobbing in the water. First turn one lamp on and observe the light distribution on a white card when placed a few feet from the lamp. Now turn the second light on such that both illuminate the card.

Question 7.14 Do you see any new dark points or lines that could be signs of interference?

Question 7.15 Based on this experiment how would you obtain the total power flow present at a point on the card from the power flow from each lamp?

Question 7.16 Does this experiment suggest a wave-like character for light?

Question 7.17 What about a model where particles are streaming out from the two lamps?

Before we adopt a particle picture, let us reconsider and see if there is a chance that we are being misled. How close does your light source resemble the two bobbing sticks? We know that the white light from the filament if sent through a prism displays a spectrum of colors. We see nothing analogous to this in our water experiment. Secondly, the light emitted from the filament is produced by the filament being heated. We might suspect that this emission is in some sense random.

Question 7.18 What would happen in our bobbing sticks experiment if they oscillated with a randomly changing phase difference? Give some careful thought to this question.

To get around this problem with the randomness of the light sources we will use a clever trick discovered in 1802 by Thomas Young.[1] Let us start with one lamp then at some distance from it we will take an opaque material and make two slits in it. Now even though the light from the source may not be regular and well

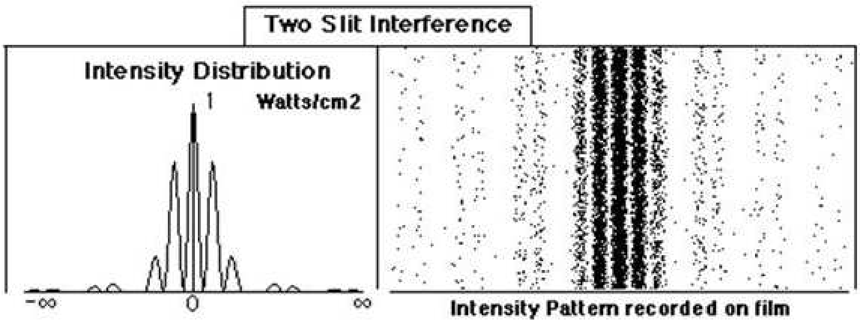
[1] Young was an English Renaissance man who studied vision, light, solid mechanics, energy, physiology, language, musical harmony, and Egyptology, according to Wikipedia. We remember him today for Young's Equation, which describes the contact angle of a liquid drop.

behaved, whatever happens to the light coming from one slit will also be happening to the light from the second slit.

Question 7.19 Is this obvious? Describe your objections.

What Young found was as long as the two slits were not very far apart his assumption was correct. While Young's trick doesn't solve the color problem, it does insure the light from our two sources (in this case, the slits) will be in phase. The color problem, if necessary, could be solved with filters.

Question 7.20 Discuss how the two pictures below are related.



End of Pre-Lab

7.3 In-class experiments on light

Find the smallest double slit on the slit film. Hold this double slit up to your eye and observe a distant showcase lamp.

Question 7.21 What do you see?

Question 7.22 Draw the light intensity you see. Is it what you expected? Is there evidence of interference?

Before we proceed with our investigation of the double slit you might like to look at the pattern produced by only one of the slits. Find a single slit on the slit film that is same width as one of your double slits.

Question 7.23 Is the pattern of the showcase lamp what you expected?

Do the observations support a wave or particle picture? Lets spend a little time comparing the single and double slit patterns. Observe the lamp with the single and four separate double slits, each of the double slits has the same width as the single slit.

Question 7.24 Draw the intensity pattern of the single slit; Directly below it draw the double slit pattern for general slit spacings. Keep the same horizontal scale for both drawings. Use the single and double slits at the middle of the slit film.

7.3.1 Location of Maxima and Minima

In the diagram, the screen is so distant that r_1 and r_2 are parallel.

Question 7.25 If the light from each source travels a different distance and this difference is equal to one-half a wavelength, what will an observer see at P?

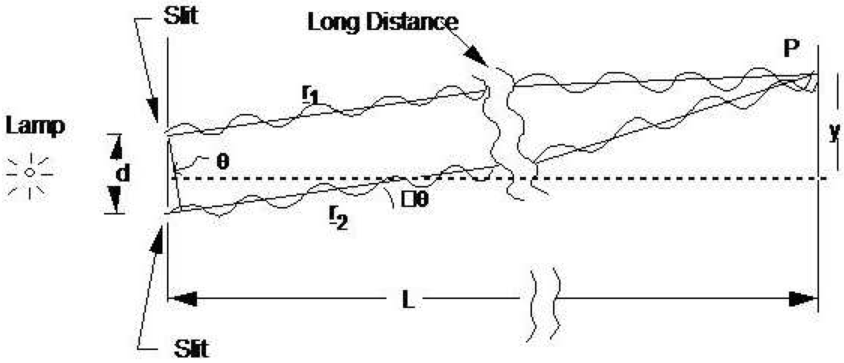


Fig. 5

Question 7.26 If the path difference ($d \sin\theta$) is one wavelength, will you see constructive or destructive interference at P?

$$\text{Condition for Maxima:} \qquad d\sin\theta = n\lambda, \qquad n = 0, 1, 2, 3, \ldots \tag{7.2}$$

$$\text{Condition for Minima:} \qquad d\sin\theta = (n + 1/2)\lambda, \qquad n = 0, 1, 2, 3, \ldots \tag{7.3}$$

where you can use the small angle approximation to get

$$\sin\theta = \frac{y}{\sqrt{L^2 + y^2}} \approx \frac{y}{L}. \tag{7.4}$$

We have glossed over an important point in this explanation, why doesn't the light after passing through a slit just proceed in a straight path casting a sharp and exact image of the slit on the screen? Clearly that did not happen in the single slit on the graphite slide.

The explanation is that waves obey Huyghen's Principle (pronounced Eye-ghen). This principle states that each point on a wave behaves as a source of a new wave and the new wave front is constructed from the tangents to these many wave surfaces. See Fig. 6. Point on the edge of the wave front from the slit spill over into shadow region.

To compare the patterns with light of different wavelengths, bring out the ancient Pepsi can covered with gels and sit it on the showcase lamp. Now look at this with your slit film. Note the locations of maxima and minima for the colors.

Question 7.27 Give an explanation for the locations using Fig. 5.

7.3.2 Wavelength Measurement of Laser Light

The wavelength of light can be determined by the diffraction pattern. The ingredients are the conditions for either the maxima or minima, the fringe spacing, and distances in the system.

Set the laser to shine through a double slit and onto a screen.

Question 7.28 First, manipulate the equations for either the maxima or minima to get λ in terms of the fringe spacing, $y_n - y_{n-1}$.

Question 7.29 Does the fringe spacing remain constant?

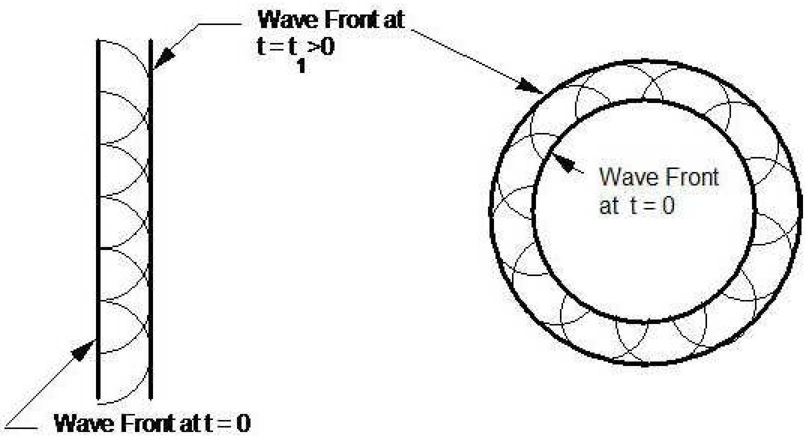


Fig. 6 Huygen's Principle

Question 7.30 If so, how can you use this to improve the accuracy of your measurement?

Question 7.31 Determine the wavelength of the laser's light from the equation you found in problem 7.28. Compare your results with that available from the laboratory instructor.

Question 7.32 Discuss the source of errors in the measurement.

7.3.3 Single Slit Pattern

Before moving on to more slits, let us investigate in detail the pattern of a single slit. Observe the lamp through the various single slits on the slit film. Describe the behavior of the pattern as the slit is made wider. Use (1,1), (1,2), (1,4), (1,8), (1,16), and (1,32). How does the fringe spacing change?

Question 7.33 How can we understand the presence of maxima and minima for the single slit?

In the lecture, you will compute the locations for each minimum. A very important result from this computation relates slit width and wavelength to the angle between central maxima and its first minima on its side.

$$\theta = \frac{\lambda}{D} \tag{7.5}$$

We will return to this equation several times in the next week.

The single slit pattern is called a diffraction pattern. It is customary to talk of interference when only a few signals are present and to reserve the term diffraction for situations where a large number of signals must be considered.

Question 7.34 If you were told to study the detailed structure of the filament from far away and were forced to look through one of the five single slits on the film, which would you use and why?

Place two of the showcase lamps in a corner of the room as close as possible to each other. Now view them with different single slits, be sure and use also the narrow slit made on the microscope slide. Determine the distance at which the two lamps are resolvable with the narrowest single slit on the slit film. In order to understand what is going on, let us draw the pattern of the two sources on the retina.

We have drawn the images overlapping in Fig. 8.

Question 7.35 As you move closer to the sources, is the overlapping increased or decreased?

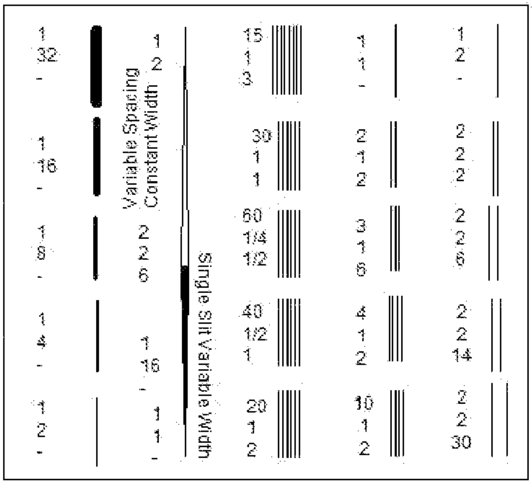


Figure 7.4: Here is the schematic of the slit film and the arrangement of single slits, double slits, multiple slits, etc. on the viewing slide. Beside each element, the top digit refers to the number of lines in the element in the finished slide, the middle digit to the width in points of the elements before the $8\times$ reduction and the bottom digit to the spacing in points between lines. One point is 0.013837 inch, a measure commonly used by printers, which was the basis for the original chart. To convert into useful units, the measure given on the slit film can be multiplied by 43.9325 micrometers.

Clearly, to resolve the two sources, it is important that the overlapping is not too great. Use the small slit on the slit film, view the two lamps from the other end of the room, move toward the lamps and try to arrive at a criteria and thus a distance for resolving the two sources. When you believe the sources are just resolved have someone cover Source 1 and note the position of its first minima and compare with the location of the central maximum of Source 2. Make a rough measurement of the angular separation of the two lamps from your position where you have just resolved the two sources.

Recall the width of a single slit pattern from equation (7.5). This would be about as close as the two lamps could be to each other with you still being able to tell they were distinct and separate.

Question 7.36 What would the angular separation between Source 1 and Source 2 have to be in order for the first minimum of one source to fall on the central maximum of second? Compare the angle $\lambda/D_{(\text{slit width})}$ with the previously measured angular separation between the two lamps.

Question 7.37 Try a larger slit; Does the resolution improve or become poorer?

7.3.4 Resolution of Your Eye

In the room make two marks close together (with a few millimeters separation) on the blackboard or on a piece of paper attached to the wall. Slowly back up until you can no longer resolve these lines.

Question 7.38 Make the measurements necessary to determine the minimum angle you can resolve.

Question 7.39 Compare with the diffraction limited prediction for your eye.

Question 7.40 Do astronomers use large diameter telescopes only for the increased light gathering ability? Discuss.

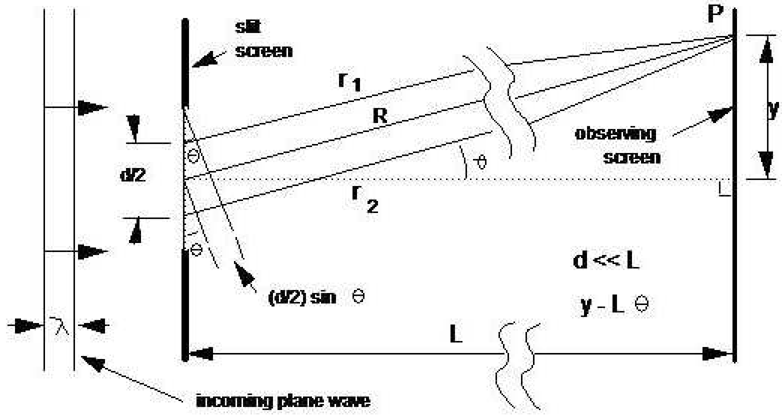
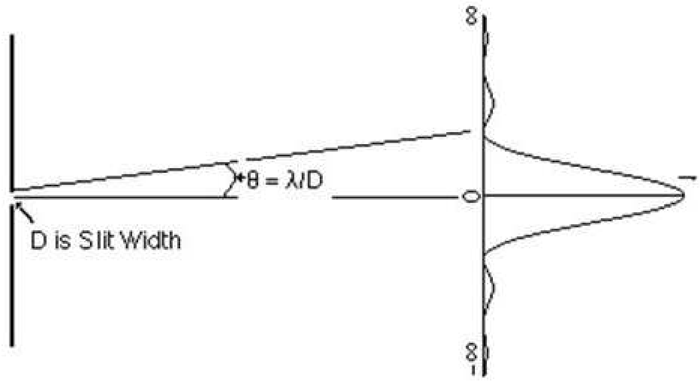


Fig.7 Geometry of Interference from a Single Slit

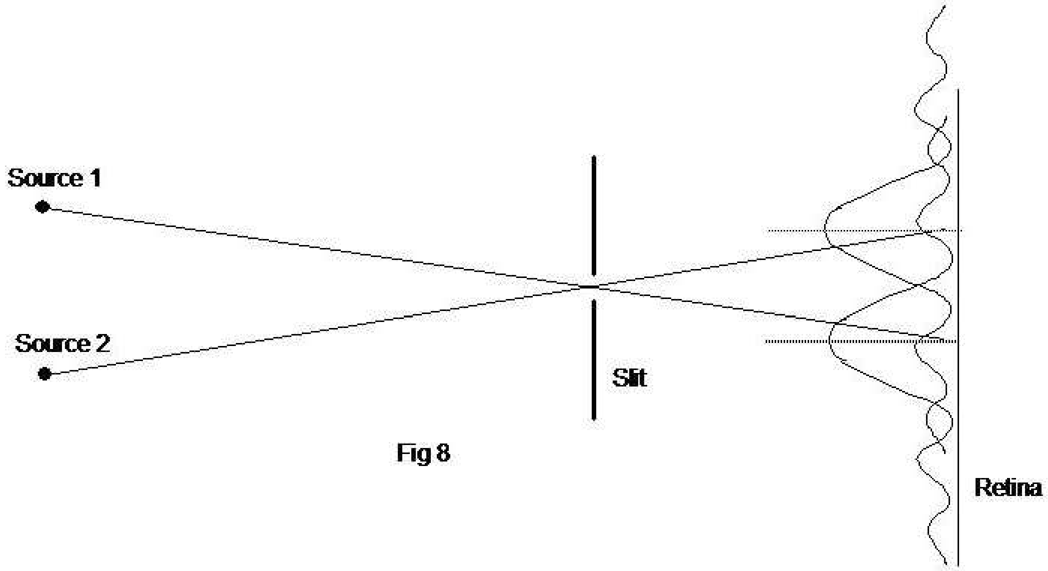


7.3.5 Diffraction from Circular Apertures & Objects

We have studied diffraction by slits in some detail, because of time limitations the extension to circular apertures will be brief. You begin by observing the light source through a variety of pinholes. Again we note diffraction patterns and suspect that similar arguments made about resolution, etc. will be true for the hole. Here we would have to divide our hole into rings and sum the contribution from each at our observation point. When this is done we find the angular width from the center to the first minimum is $\Delta\theta = 1.22\frac{\lambda}{a}$ where a is the aperture diameter and λ is wavelength of light.

Question 7.41 Use this result to determine the diameter of one of the smaller pinholes.

Once you are familiar with the pattern due to a hole, it is possible to draw some conclusions about the pattern produced by light falling on an obstacle. Consider a plane wave falling on a hole in an opaque object. Next we plug the hole, the field present at the screen E_{screen} is zero, remember we are now behind a fully opaque object. We can also write $E_{screen} = E_A + E_S + E_{plug} = 0$ where E_A is the field at the screen from the object with a hole in it and E_{plug} (the field from just the plug). Our field on the screen S with the plug removed is



$$E_A = E_{screen} - E_{plug} = 0 - E_{plug} = -E_{plug}$$

Thus the pattern for the obstacle should be the same as that for the hole.

Question 7.42 Why don't we worry about the negative sign?

The first zero occurs at the same angle as before $\Delta\theta = 1.22\frac{\lambda}{a}$ where a is the obstacle diameter.

Stop first lab session

Start new lab session

7.4 Interference using the Computer Interface

7.4.1 Equipment List

1. Science Workshop 750 Interface Box
2. Light Sensor, Aperture Bracket, Rotary Sensor and Linear Translator Apparatus
3. Flat Optical Bench
4. Diode Laser
5. Multiple Slit Set and Slit Accessory Holder

7.4.2 Introduction

In this experiment you will use a Light Sensor to record the pattern from single and and double slits on computer using the Data Studio Interface. Your data will be plots of Intensity versus Position.

7.4.3 Experimental Setup

Connect the Science workshop Box to the computer, turn on the interface and the computer.

Connect the light sensor into analog channel A on the interface.

Connect the rotary motion sensor stereo phone plugs into Digital Channels 1 and 2.

Open a new instance of Data Studio.

Set up a Graph display of Light Intensity vs. Position. You can click on the x axis to change it to position. The data recording should be set at 50 measurements per second (50 Hz)

Mount the diode laser on one end of flat optical bench. Connect the power supply to the laser.

Place the multiple slit set into the slit accessory holder. Mount the slit accessory holder in front of the diode laser on the bench.

Mount the translator apparatus on the optical bench. If necessary remove the O-ring and thumbscrew from the rotary motion sensor pulley so they will not interfere with the aperture bracket.

Turn on the power switch on the back of the Diode Laser. Adjust the position of the laser and the multiple slit set on the slit accessory so that the laser beam passes through on the of double-slit pairs on the slit set and forms a clear, horizontal diffraction pattern on the white screen of the aperture bracket.

Record the slit width a and the slit spacing d of the double slit you selected.

Rotate the aperture disk on the front of the aperture bracket until the narrowest slit is in the front of the Light Sensor opening.

You will not need to calibrate your sensor. You will, however, have to find the conversion ratio from degrees in rotary motion measured from the rotary motion sensor into length the sensor moves in the horizontal axis. This can be done by measuring the rotary position, moving the sensor into a new location, and determining the distance it moved and the corresponding change in rotation.

7.4.4 Getting Comfortable

Before recording any data for analysis you should experiment with the rotary motion sensor and light sensor setup. Gently move by hand the rotary motion sensor/light sensor along the rack until the maximum at one edge of the diffraction pattern is next to the slit in front of the light sensor.

Begin recording data by clicking the start button on your screen. slowly and as smoothly as possible, move the rotary motion sensor/light sensor assembly so that the slit of the aperture disk moves across all the maxima of the diffraction pattern. When the entire diffraction pattern has been measured then stop recording data by clicking the start/stop button.

Examine your data, make a rough estimate of the laser light wavelength.
Erase your data.

7.4.5 The Double Slit

Move the rotary motion sensor/light sensor along the rack until the maximum at one edge of the diffraction pattern is next to the slit in front of the light sensor.

Click the start button to begin recording data.

Slowly and as smoothly as possible, move the rotary motion sensor/light sensor so that the slit of the aperture disk moves across all the maxima of the diffraction pattern. When the entire diffraction pattern has been measured then stop recording data by clicking the start/stop button.

7.4.6 The Single Slit

Replace the multiple slit set with the single slit set on the slit accessory holder. Select a single slit. Record it's width.

Re-align the laser beam with the selected single slit on the single slit set. Adjust the positions of the laser and the rotary motion sensor, if necessary, so that the diffraction pattern is at the same height as for the double-slit.

Repeat the procedure for collecting data that you used in the double-slit measurement. Select several different slit widths and observe the pattern.

7.4.7 Final Analysis of the Data

Question 7.43 Draw the shapes of the plots of light intensity versus position for both the double-slit and the single-slit patterns. Discuss similarities and differences.

Question 7.44 Do you see any remnant of the single slit pattern in the double slit patterns?

Question 7.45 From your double-slit results determine the laser light wavelength. Compare with the value given on the instrument.

Question 7.46 From your single-slit results, determine the angular width of the central maximum, $\Delta\theta$.

The angular width is one half the angular width between the first zero intensity points on opposite sides of the central maximum. The angle is measured from the slit set to the aperture slit.

Question 7.47 Compare your results with the predicted value of $\Delta\theta = \frac{\lambda}{D}$ where D is the slit width.

Question 7.48 Extra Credit If you have the time and wherewithal, take the double slit intensity data and do a Fourier transform with respect to the distance in order to find the fringe spacing. Use this to calculate the wavelength.

EXPERIMENT 8

Diffraction Grating Spectroscopy

8.1 Equipment Required

1. Sodium lamp
2. Double Slits
3. Spectrometer
4. Diffraction Grating
5. Slit Film
6. Mercury Arc Lamp
7. Black Cloth

8.2 Introduction

You are now familiar with the important patterns expected from single and double slits. These patterns not only demonstrate the phenomena of diffraction and interference which suggest to us that light is wavelike in its behavior; they also permit the determination of the wavelength. Let us determine the wavelength of the light emitted by a sodium lamp.

Try to do this as you did in §7.3.2, using just a double slit.

Determine the slits spacing with the traveling microscope. Is it clear that a device that would enable you to see the fringes and directly measure the angle of various fringes as they leave the slits would greatly facilitate the measurement? There exists such a device; it is called a spectrometer.

8.2.1 Getting to Know the Spectrometer

Obtain a spectrometer and study its construction. There are four rather obvious parts: 1) something that looks like a telescope, 2) another tube that is similar, but with an adjustable slit on one end (precision slits are difficult to make and expensive), 3) a circular table (stage) with a hold-down bracket, and 4) a large graduated circular table with a vernier system. Give a brief overview of the spectrometer and the light path through the spectrometer.

The function of each item is clear if we remember what it is we want to do. We first would like to mount our double slit (item 3). Next we would like to illuminate our two slits with light from a distant source so that parallel light falls on the slits. Item 2 does this for us and is called a collimator. When properly adjusted, light from our source (the sodium lamp) falls on the adjustable slit which is placed at the focal point of the objective lens of the collimator.

Question 8.1 What is the nature of the light coming out of the collimator? (Hint: what does a collimator do? If you do not know, needle the TA)

Question 8.2 At what distance is the adjustable slit imaged?

Now that we have parallel light on the double slit we expect an interference pattern; move the telescope out of the way and observe the pattern with your eye. The graduated circle (item 4) and the telescope (item 1) will permit you to measure the angular position of each fringe. Make sure your double slit is at the center of the graduated circle.

Since the incoming light is parallel you should focus your telescope at infinity by adjusting the eyepiece. To do this you can carefully take it to the corridor and focus on a distant object.

Note that we have cross hairs in the scope in order to accurately locate the center of a fringe. We need the cross hairs and the fringe image to both be clearly in focus when viewed through the eyepiece. This means the cross hairs and the image due to the objective must be in the same position. While focused on a distant object, move the cross hairs until they are also in focus.

Now move your head slightly from side to side (using the method of parallax) to ensure both are at the same position. Don't move the cross hairs relative to the objective again; you may, however, adjust the eyepiece to suit your vision. Once the telescope is focused, it is trivial to focus the collimator objective on its slit. Simply illuminate the slit and view the image through the telescope.

Adjust the collimator objective until the image seen in scope is in focus with the cross hairs. Remember since the scope is focused to infinity, only parallel light entering will be in focus.

Investigate the graduated circle of vernier; make sure you understand its operation. If you discover you do not, then bring over the TA.

Question 8.3 Measure the wavelength of the yellow line of sodium using a double slit in the spectrometer. What did you find?

Before proceeding, place your slit film in the light from the collimator and observe the variety of patterns through the telescope from the double slits.

Question 8.4 Compare your wavelength measurement with a value obtained from the laboratory instructor. If you are off by more than 5%, check for errors.

Question 8.5 If we add more slits, will there be more or less destructive interference?

Observe a narrow light source through the Slit Film with your eye. Use the slits from the column where only the number of slits is changed (2,1,2), (4,1,2), (10,1,2), (20,1,2). The slit width and separation should remain the same.

Question 8.6 What change do you see actually see?

Question 8.7 Draw the intensity pattern for all five cases.

Let's try to understand what is happening.

8.3 Extending the Double Slit to Many Slits

When δ the path difference to point P is an integral multiple of λ, we get a maximum. When δ is an odd multiple of $\frac{\lambda}{2}$, we get a minimum.

Again when δ is an integral multiple of λ we should get a maximum again. Check your drawings and confirm that indeed the maximum stays fixed as N, the number of slits changes. Remember that d remains constant. We also will find a minimum when δ is an odd integral multiple of $\lambda/2$.

For the next few problems, consider the light coming from two and four slits, as envisioned in Figures 8.1 and 8.2

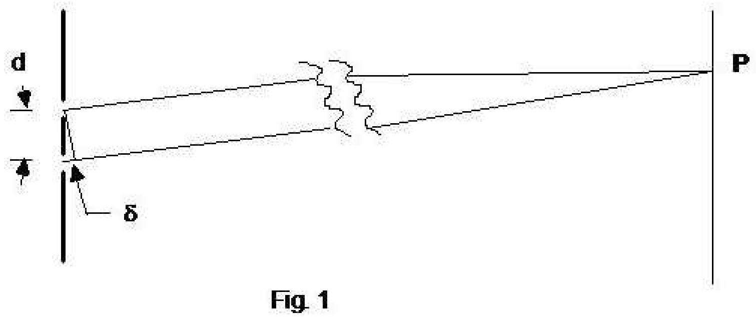


Figure 8.1: Two slit path differences

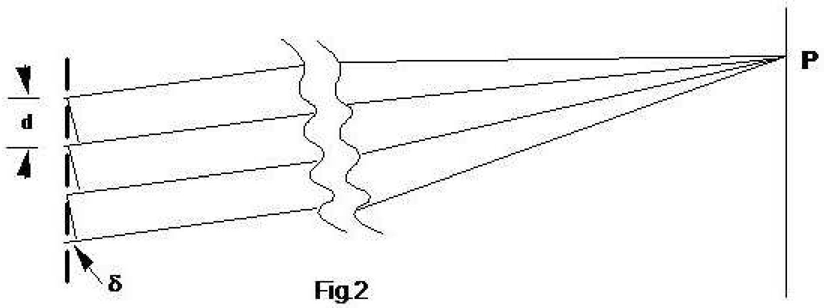


Figure 8.2: Four slit path differences

Question 8.8 What should we find if $\delta = \lambda/4$?

Question 8.9 Draw the contribution at P from each slit versus time at the slits and at the point P in the case of Figure 8.2 with a path difference $\delta = \lambda/4$.

Thus we have picked up a zero at $d \sin\theta_{min} = \lambda/4$ which was not present in the $N = 2$ case. Consider $N = 3$ and $N = 10$.

Question 8.10 Discuss how the numbers of maxima and minima change as N increases.

Question 8.11 What would happen if N were of the order of 15,000, such as in your diffraction grating?

Take such a diffraction grating and place it on your spectrometer. Now determine the wavelength of the sodium yellow light.

Question 8.12 Do this in first and second order, i.e. at angles where $\delta = \lambda$ and $\delta = 2\lambda$. This second order measurement is difficult since the line is so weak. You should darken the room and place black cloth or paper around the source so that no stray light gets into the telescope.

Question 8.13 What unusual thing do you notice about the second order fringe that you never saw with $N = 2, 4, 10$, or 20? You can also see it in first order. Be sure your slit is adjusted as narrowly as possible and that the image is in focus.

Obtain an intense mercury lamp and observe its higher orders. Note the separation between lines as you move to higher order.

Question 8.14 Why don't we always just work in higher order?

End First Session

Start New Session

In the following you will learn of some of the other characteristics which determine the quality of a grating spectrometer.

8.4 Diffraction Grating Spectroscopy

Comment: We have seen that by using many slits, it is possible to resolve two wavelengths which are close together. The grating used had 15,000 lines/inch. Reflect for a moment on how you might construct such a grating.[1] While we have exclusively discussed transmission gratings, let it be clear that a regular array of highly reflecting lines on a transparent or opaque background would produce the same pattern.

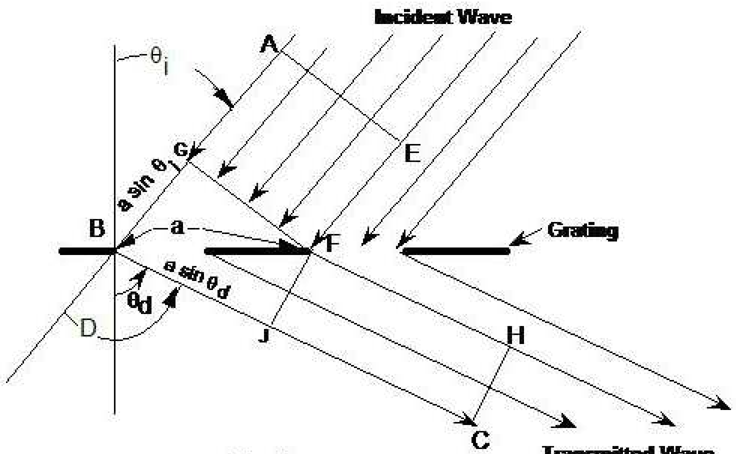


Fig. 3

Let us re-derive our grating equation for the case of a plane wave incident at an angle θ_i (Fig. 3). Consider waves emerging in the direction θ_d; in particular, the rays **ABC** and **EFH**. How these two rays interfere when they are focused will depend on their path difference. From Fig. 3 we define this difference δ, which is

$$\delta = a \sin\theta_d + a \sin\theta_i \tag{8.1}$$

If this path difference is $\delta = n\lambda$ then when the two rays intersect they will constructively interfere and produce a maximum at some angle $\theta_d = \theta_m$. This is true for all pairs of rays from corresponding points of the grating.

The *Grating Equation* is

$$a \sin\theta_m + a \sin\theta_i = n\lambda. \tag{8.2}$$

This quite simple equation can be used to good advantage.

8.4.1 Minimum Deviation

In Fig. 3 we see that the incident light has deviated from its original direction. This angle of deviation D is:

$$D = \theta_i + \theta_m$$

[1]See A.R. Ingalls, *Scientific American* 186, 45 (1952) or E.W. Palmer and J.F. Verrill, *Contemporary Physics* 9, 257 (1968).

Using a trigonometric identity we can rewrite equation (8.2) as

$$n\lambda = 2a \sin\left(\frac{\theta_i + \theta_m}{2}\right) \cos\left(\frac{\theta_i - \theta_m}{2}\right)$$

Introducing D we can write

$$n\lambda = 2a \sin(D/2) \cos\left(\frac{\theta_i - \theta_m}{2}\right) \tag{8.3}$$

Question 8.15 We can use this equation to measure λ. How do we do this experimentally?

Question 8.16 Use the mercury light source, pick a bright line and determine λ using this method. Record your value.

Presumably you had some difficulty determining θ_i and θ_m.

Question 8.17 Which quantity in your calculation has the largest percentage error (similar to the relative error), $\theta_i + \theta_m$ or $\theta_i - \theta_m$? As an example, take $\theta_i = 10 \pm \frac{1}{2}$ and $\theta_m = 9 \pm \frac{1}{2}$. Explain how you arrived at this conclusion.

Question 8.18 Where is the cosine above the least sensitive to errors in $\theta_i - \theta_m$? To do this, use the standard method for computing uncertainty and find where it is at a minimum.

Question 8.19 Show that D has a minimum when $\theta_i - \theta_m = 0$. This minimum leads to the equation

$$\sin\left(\frac{D_{min}}{2}\right) = \frac{n\lambda}{2a}. \tag{8.4}$$

Because of this, if we could rotate our grating and find the position of minimum deviation and measure D_{min}, then λ could be computed quite accurately.

Using the mercury lamp as a source of light, adjust the grating and the telescope for minimum deviation for a given line in the spectrum; first when the total deviation is to the right and then with the total deviation to the left. One half of the difference of the two readings should give a good average value for D_{min}. Make sure the settings are for minimum deviation. Repeat this for the other wavelengths.

Question 8.20 Reproduce Table 8.1 in your lab manual and record your data and computations.

Question 8.21 Discuss your results.

λ(Å)	Order	Left	Center	D_{min}	Right	Center	D_{min}	$(D_{min})_{Av}$	$\lambda_{calc.}$	% error
4338(Violet)	1									
	2									
5461(Green)	1									
	2									
5770(Yellow)	1									
	2									
5791(Yellow)	1									
	2									

Table 8.1: The spectrum for a low pressure mercury light source. (The terms Left, Right, Center, refer to vernier position.)

EXPERIMENT 9

The Michelson Interferometer

9.1 Equipment Required

1. Michelson Interferometer
2. Sodium Light Source
3. Mercury Light Source
4. Laser
5. Laser Optics Kit

9.2 Introduction

The Michelson interferometer is a versatile instrument with a rich history. In 1887, Michelson and Morley, using a very large version, attempted to determine the motion of the earth through the ether. Their instrument had extremely long path lengths (11 meters). It was constructed on a concrete slab and floated in mercury in order to be easily rotated.

The idea behind the experiment was quite simple. Light is a wave phenomena and thus something should wave. Recall that in water waves the water moves up and down and back and forth. Naturally, the question was what moves up and down in a light wave?

It was postulated that there was a medium in which oscillations took place; this medium was called the ether and it was thought to pervade all space including a vacuum since light has no difficulty propagating through a vacuum. Michelson decided to try to detect the motion of the earth through the ether.

With the interferometer he failed to find any motion. The negative results of this experiment was one of the important facts which supported the special theory of relativity devised by Einstein in 1905. The ether experiment is in *Philosophical Magazine* 24, p. 449 (1887); it is easy reading and can be enjoyed.

Michelson conducted a number of other, now classical experiments, with his interferometers. He measured the wavelengths of cadmium red, green and blue light, and helped to define the meter. The interferometer was also used to study the fine structure of spectral lines, notably those of the red H_α line in hydrogen.

9.2.1 Interferometer Principles of Operation

The essential feature of any interference experiment is two optical paths which differ. The light arriving at a point of observation then is the superposition of both beams; their relative phase will determine the intensity observed. When in phase we expect constructive interference; when the phase difference is 180° then destructive interference leads to zero intensity. By inserting a different medium in the path of one beam we will alter the interference pattern. Analysis of the altered pattern provides information about the medium we inserted.

The two paths the light takes are shown in Figure 1. The optical components consist of two highly polished mirrors M_1 and M_2 and two glass plates P_1 and P_2. Plate P_1 has a thin silver coating on it such that a light beam falling on its surface will be partially reflected and partially transmitted towards mirrors M_1 and M_2.

After reflection from these mirrors, a portion of the light from M_1 will be transmitted by P_1 while a portion reflected by M_2 will be reflected by P_1. An observer can witness the resulting superposition at O.

In order that we clearly understand this instrument it will be necessary to do some qualitative experiments first.

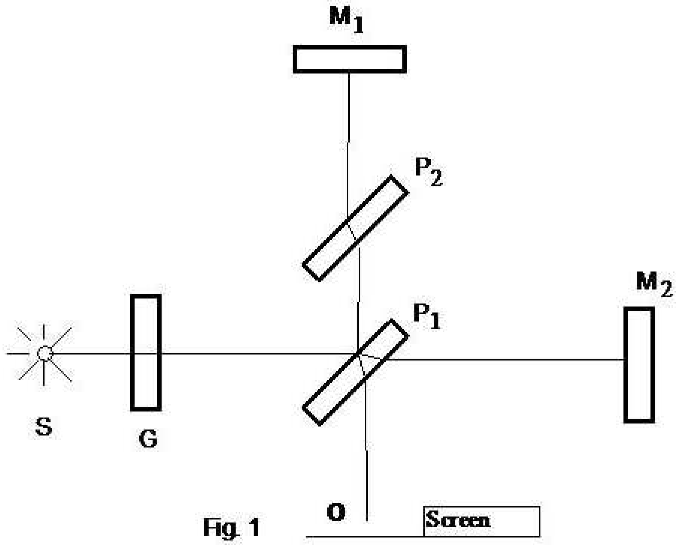


Fig. 1

9.3 Measuring Laser Wavelength

Take a laser and shine the beam into the interferometer. Remove the ground glass diffuser from its holder. Do not look directly into the interferometer.

Observe the laser beam leaving the system on a distant wall. There will ordinarily be two bright spots. There are also some other spots, see if you can find their source. Once you know where they come from you can safely ignore them. Since the two beams represent light that has traveled separate legs of the interferometer we should be able to superimpose them by adjusting one of the mirrors. One mirror has two adjustments; carefully bring the two spots together. Now slowly turn the micrometer screw and observe the spot move.

Question 9.1 What happens to the intensity when the two dots are on top of each other?

Question 9.2 How much farther in terms of λ did the beam along one leg travel than the beam along the second leg when the intensity is a minimum?

Question 9.3 How far must you move the mirror in terms of λ before another minimum appears in terms of the light's wavelength?

In order to get a rough estimate of the wavelength of the laser light, determine the distance moved by the mirror to produce 50 minima. (*Remember:* the micrometer-mirror drive produces 5-1 reduction. This means that for each millimeter the micrometer moves, the mirror moves .2 mm.)

Question 9.4 What wavelength did you find?

If it is off by a factor of two then recall that the wave goes down and back so a change of mirror position by a d will change the light path length of the light by 2d.

Adjust the spot for a minimum. Reduce the light in one leg by inserting your hand. (Never touch any of the glass or mirror surfaces.)

Question 9.5 What happens to the spot?(If it is difficult to decide use a diverging lens.)

9.4 Images and the Method of Parallax

Use the laser and a diverging lens to produce a point source. Shine this light into the interferometer and observe the pattern on a screen or the wall. There should appear dark and bright fringes which are circular if the mirror is carefully adjusted. Note that no lens is necessary to see this interference pattern.

Question 9.6 What kind of image is that—real or virtual?

Let's pause now and see what is happening. Turn the laser off. Look into the interferometer; you can clearly see an image of the adjusting mirror in the beam splitter. You also see the image of the movable mirror, however, it may not be at the same position as the other mirror. Now place a pencil at the entrance. Vary the angle of the adjustable mirror and note the two images of the pencil.

From these observations, it should be clear that for ease of understanding we can replace our interferometer by an equivalent arrangement as shown in Fig. 2.

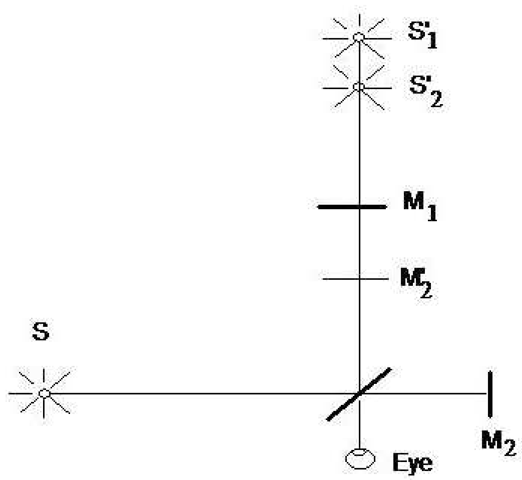


Fig. 2

Thus we see the image of S in M_1 (labeled $S_1^{'}$) and also an image of S in $M_2^{'}$ (labeled $S_2^{'}$). The two sources $S_1^{'}$ and $S_2^{'}$ are coherent.

Question 9.7 Why should the images be coherent?

Question 9.8 Draw the interference pattern to be expected from two such sources.

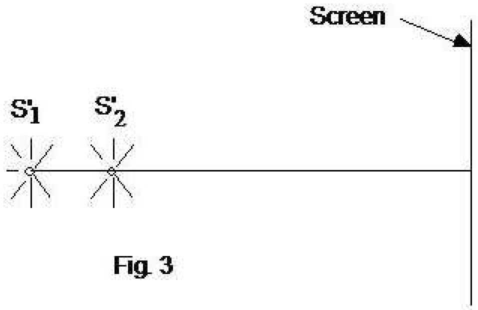


Fig. 3

Question 9.9 As a mirror is moved, what is happening to the sources?

Question 9.10 If the two mirrors are not exactly parallel, what effect will this have on the interference pattern? Illustrate.

Question 9.11 Is the pattern which appears on the screen the same as you would see if you look at the light from the two sources with your eye or used a lens to project the image on a screen?

Consider two rays which are constructively interfering. Now insert a lens and find its effect.

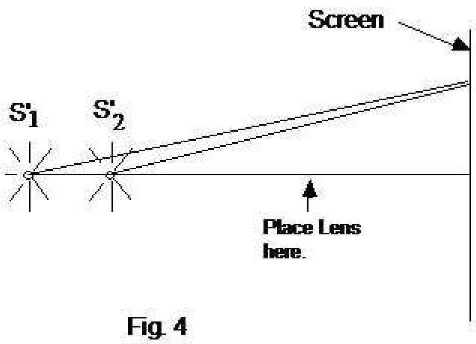


Fig. 4

Question 9.12 Draw what you expect to see in the focal plane of the lens.

Do the experiment with a lens, not your eye. Besides this not blinding you, the eye has such a small aperture that you will intersect very few rays and thus the interference pattern is difficult to see.

In summary we see that a point source of light into the interferometer produces well defined fringes. However, in order to see them they must be observed on a screen, or a lens must be used to aid the eye. In practice an extended source is used.

9.5 Extended Sources with the Interferometer

Place an extended source (mercury or sodium vapor lamps) at the input to the interferometer. Be sure you put the ground glass in its holder to provide a more uniform illumination.

Make the necessary mirror adjustments so that well-defined circular fringes are seen. Let us try to understand the source of this beautiful pattern. First, recall that since this is a finite source we can say something about the coherence of each point on the source.

Question 9.13 Are they coherent?

In Fig. 5 the two images of the source S_0 is shown; S is the image in Mirror 2 and S' is the image seen in Mirror 1.

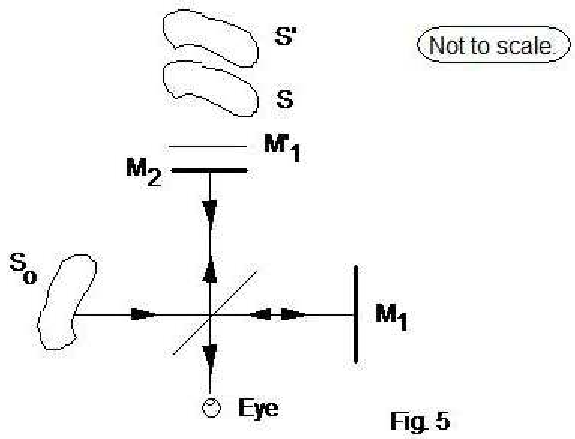


Fig. 5

Select two coherent points S_1 and S'_1. Consider the two parallel rays shown in Fig. 6, as we have seen earlier these will be brought together in the focal plane of the lens (L). Parallel rays leaving S_1 and S'_1 at another angle will be imaged at a different point on the screen. However any rays from S_2, S'_2 or S_3, S'_3, etc. which are parallel to those shown leaving S_1, S'_1 will be focused at I_1. (The rays from S_1 and S_2 in the figure are not parallel). In the diagram we show two sets of rays leaving S_3, S'_3 the dotted rays are parallel

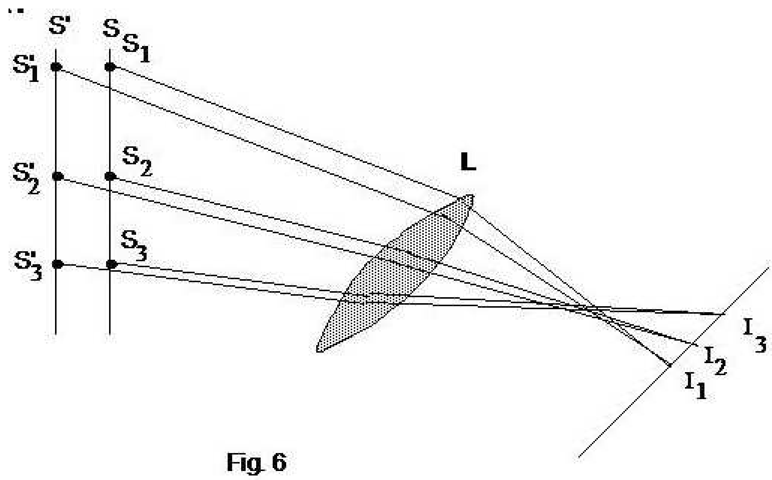


Fig. 6

to those leaving S_2, S'_2. It is important to note that if the rays from S_1, S'_1 differ by 180° upon arriving at I_1 then any parallel rays leaving S_2, S'_2 will also have a 180° phase difference and will contribute a minimum at I_1.

It should be clear from the diagram that if I_1 is a minimum, then I_2 might have a maximum since the phase between the rays from S_2, S'_2 will be different. I_3 might be another minimum. It should also be clear that if the mirrors are perpendicular then the pattern will have circular symmetry about an axis from the lens perpendicular to the mirrors.

If we investigate the two rays leaving S_1, S_1' we see that their path difference δ is

$$\delta = 2d\cos\theta \tag{9.1}$$

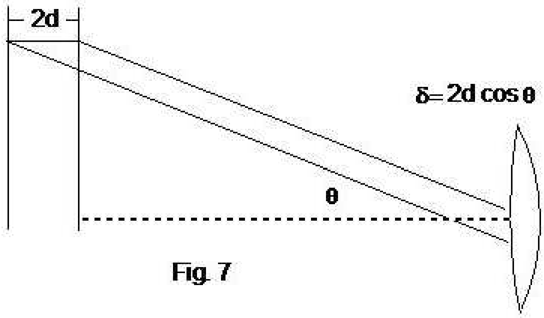


Fig. 7

where d is the actual separation of mirrors.

Question 9.14 Either draw your favorite magical creature or show that $2d$ is correct separation of virtual sources.

Question 9.15 Doesn't the lens affect their phase difference? They do go through different parts of the lens.

Question 9.16 Draw a diagram of a lens with the two rays and try to give a rough qualitative argument that the lens might not introduce any phase difference.

It should be pointed out that the path difference δ may not be the only phase difference present. Let us follow each ray through the interferometer (Fig. 7).

A ray leaves S_0 and is divided by the beam splitter. The reflected light first passes through a glass plate (the metal coating is on the back of the plate) and is then reflected. It then reflects from M_2 and passes through the beam splitter again. Note it has made three passes through the glass plate.

The other ray passes through the beam splitter, reflects from M_1 then reflects off the back side of the beam splitter.

It would have traveled only once through the glass plate and this would introduce a phase difference; however, as shown in the diagram, a second glass plate (the compensating plate is inserted to equalize the optical paths. Some beam splitters use a piece of uncoated glass, in these cases, one beam suffers internal reflection with a reflection coefficient of $\approx (n_g - n_{Ai})/(n_g + n_{Ai})$, while the other beam is externally reflected. The reflection coefficient for this is $\approx (n_{Ai} - n_g)/(n_g + n_{Ai})$. Thus there is an additional 180° phase shift introduced.

As you move the micrometer, verify that the fringes disappear or reappear in the center depending on the direction in which the mirrors are moved.

Question 9.17 See if you can give an argument with a diagram which explains this behavior.

Question 9.18 How far in terms of the wavelength must you move a mirror before a dark fringe at the center of the pattern is replaced by another dark fringe?

Appendix A

The Helium-Neon Laser

The continuously operating gas laser depends on the following principles:

1. Many atoms in a low-pressure gas carrying an electrical discharge become "excited" by collisions to electronic arrangements different from the ground state that ordinarily exists in the absence of the discharge. These excited states have sharply defined energy values, greater than that of the ground state.
2. The probable time of existence of some excited electronic states(metastable states) is relatively long (of the order of milliseconds, compared to more usual lifetimes of less than a microsecond).
3. An atom can go from a state of higher energy to one of lower energy by emitting a photon of frequency $\nu = \Delta E/h$, where ΔE is the energy change, and $h = 6.63 \times 10^{-27}$ erg-sec is a universal natural constant, Planck's constant.
4. The presence of light at frequency ν encourages the transition. (Photon emission occurring in response to a light signal is called stimulated emission, in contrast to randomly occurring spontaneous emission.)
5. Stimulated emission occurs with a precise and reproducible phase relation to the light signal that caused it.
6. One can obtain conditions where the emission from metastable atoms is largely of the stimulated type.

In the simple He-Ne laser, a continuous electrical discharge causes, among many other things, the creation of metastable neon at such an energy level that the photons emitted upon transition to a lower state have a wavelength of 632.8 nm. Accurately aligned mirrors near the ends of the discharge tube reflect plane waves of light back and forth to produce standing waves. Emission of a 632.8 nm photon by a metastable neon atom starts the buildup of a standing wave at the corresponding frequency. The standing wave in turn produces stimulated emission from other metastable atoms, always synchronized so as to reinforce the standing wave. An equilibrium is reached such that the rate of creation of metastable atoms by the discharge equals the rate of loss by stimulated emission. A portion of the light escapes as a plane wave through one of the mirrors, which is not coated heavily enough to give complete reflection.

The oldest lasers used in the laboratory have a light output of about .6 milliwatt. Within the small diameter of the beam (between 1 and 3 mm) this represents almost half the intensity (power per unit area) of sunlight (all wavelengths included). The beam spreads at large distances into a cone of full angle 8 milliradians, only twice the theoretical (diffraction limited) value for a beam starting with a 11.5 mm diameter.

The discharge tube draws about 7 milliamperes, and operates at a voltage drop (dc) of 1600 V. For stable operation, the tube must be connected through a voltage-dropping resistor to a supply of higher voltage. The smallest supply voltage used is about 2400 V. The supplies using this voltage contain a starting circuit which adds a brief voltage pulse of perhaps 1000 V. Other supplies have a voltage of 3000 V or more, and require no pulse to start the discharge.